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MARMOTTE
A Multinational Model
By CEPII / CEPREMAP

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ABSTRACT

MARMOTTE is an annual multinational model of 17 OECD countries focussing on the medium term. Each country is modelled by a system of about 50 equations. This rational expectations model has strong microeconomic foundations as most of the behavioural equations result from inter-temporal optimisation. Due to these characteristics, the simulation results are easy to interpret. This model is currently used under the framework of a European Network to investigate the international transmission of shocks as well as to analyse the consequences of economic policy in the euro area.

MARMOTTE est un modèle annuel multinational de moyen terme. Il inclut 17 pays de l'OCDE, chacun d'entre eux étant modélisé à partir d'un système de 50 équations environ. Ce modèle, à anticipations rationnelles, a des fondements micro-économiques forts, la plupart des comportements résultant d'une optimisation intertemporelle. De par ses caractéristiques, les résultats de simulations de MARMOTTE sont facilement interprétables. Ce modèle est actuellement utilisé dans le cadre d'un réseau européen d'instituts de recherche pour analyser la transmission internationale des chocs ainsi que les conséquences de la politique économique dans la zone euro.

keywords: Dynamic general equilibrium models, perfect foresight, putty-clay technology, macro econometric modelling.

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RÉSUMÉ

Ce document présente MARMOTTE, le modèle annuel multinational du CEPII et du CEPREMAP. Ce modèle inclut les principaux pays de l'OCDE : les 14 pays membres de l'Union Européenne, les Etats Unis, le Japon et le Canada. MARMOTTE fournit une description détaillée de ces économies, en tenant compte des asymétries structurelles et propose une modélisation des interdépendances par le commerce et les flux financiers.

C'est donc un instrument approprié pour analyser la transmission internationale des chocs de différentes natures et les conséquences macro-économiques à court et moyen terme des politiques économiques.

Le modèle MARMOTTE, à anticipations rationnelles, a des fondements théoriques forts, dans la mesure où les plupart des comportements résultent d'une optimisation intertemporelle. Chaque pays est modélisé à partir de 50 équations, dont les paramètres peuvent différer entre pays. La plupart sont estimés par des méthodes économétriques récentes, tandis que les autres, notamment ceux du bloc d'offre, sont calibrés. Dans Marmotte, les pays diffèrent par leur taille, par leur degré d'ouverture, par leur régime de change et par la valeur des paramètres structuraux de leur économie.

Le document est organisé comme suit:

Le chapitre 1 expose la caractéristique la plus originale de MARMOTTE : le bloc d'offre est modélisé par une technologie putty-clay. Chaque année, une nouvelle génération de capital est installée, dont l'intensité capitalistique dépend de la technologie disponible et reste inchangée jusqu'à son déclassement. Cette technologie est particulièrement bien adaptée à l'analyse des changement de la répartition salaire - profit dans le revenu national à moyen terme.

Le chapitre 2 décrit une autre caractéristique de Marmotte : les consommateurs optimisent de manière intertemporelle une fonction d'utilité non séparable, qui prend en compte la formation d'habitude. Ceci introduit une certaine rigidité dans les comportements de consommation.

Le chapitre 3 présente les équations relatives aux identités comptables, à la courbe de salaire (la pseudo offre de travail), au commerce international et à la partie monétaire et financière du modèle. Ces équations sont relativement standards.

Le chapitre 4 expose la méthodologie utilisée pour évaluer les conditions de stabilité des modèles non linéaires à anticipations rationnelles et présente ensuite en détail la technique de simulation propre à MARMOTTE.

Dans le chapitre 5, des simulations de Marmotte permettent d'évaluer les réponses différenciées des pays européens à des chocs standards de demande et d'offre.

SUMMARY

This document presents MARMOTTE, the annual multinational model of CEPII and CEPREMAP. This model includes the main OECD countries : the 14 members of the European Union (Luxembourg and Belgium are merged), the United States, Japan and Canada. MARMOTTE provides both a detailed description of these economies, including significant structural asymmetries, and a comprehensive modelling of interdependence, through trade and capital flows. It is thus an appropriate tool to investigate the international transmission of shocks of a different nature as well as the short and medium run consequences of economic policy.

MARMOTTE has strong microeconomic foundations as most of the behavioural equations result from inter-temporal optimisation. Another feature is that the economic agents have perfect foresight. Each country is modeled by about 50 equations, with parameters that may differ between countries. Many are estimated by recent econometric methods, while others, in particular those of supply-side equations, are calibrated. MARMOTTE allows for four kinds of differences between countries : size and ratios of national macroeconomic aggregates ; openness and the geography of the trade flows; the exchange rate regime and the structure of the economy.

The document is organised as follows.

Chapter 1 exposes the most original feature of MARMOTTE: the supply side is modeled by a putty-clay technology. Each year, a new vintage of capital is installed, whose capital-labor ratio is chosen from the technology available and remains unchanged until its scrapping date. This technology is especially adapted to the analysis of medium term changes in the allocation of national income to wages and profits.

Chapter 2 describes another specific feature of MARMOTTE : consumers optimize inter-temporally a non-separable utility function, which takes into account the formation of habits. This introduces some stickiness in consumption behavior.

Chapter 3 successively presents the equations related to the accounting identities, the wage curve (the pseudo-supply of labour), the foreign trade and the monetary and exchange rate parts of the model; these equation are quite classical.

Chapter 4 explains the methodology used to assess the stability conditions of non linear models with rational expectations and presents in details the technique used to simulate MARMOTTE.

Chapter 5 comments some basic simulations of MARMOTTE and especially assesses the differentiated responses of European countries to standard demand and supply shocks.

Marmotte
A Multinational Model

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At the end of 1997 Pierre-Alain Muet, then adviser of the Prime Minister of France and Jean-Claude Berthélémy, Director of Cepii, discussed the project of building a new multinational model at Cepii. Cepii was already associated with Ofce in the use and the maintenance of a multinational model called Mimosa. Mimosa is a macro-econometric model with strong traditional Keynesian flavour. The idea was to leave the responsibility of Mimosa to Ofce, and to ask Cepii to build another model with a special focus on the supply-side of the economy. This model would have a strong theoretical background, with optimising behaviour and rational expectations. Also, the use of econometrics to estimate equations, and the research of a good fit of the model on historical data, would be less systematic than for other models such as Mimosa. So, the philosophy of this new model should have much in common with the model of the G7 countries built by the IMF, Multimod Mark 3.

The Cepii asked Pierre-Yves Hénin, Head of Cepremap, for co-operation between both institutions, which would include to give Professor Jean-Pierre Laffargue, a senior economist in Cepremap the scientific responsibility of the project. An agreement between Cepii and Cepremap was quickly reached, and a feasibility study of the project started at the beginning of 1998. The following economists have been involved in the building of the model: Loïc Cadiou, Stéphane Dées, Stéphanie Guichard, Arjan Kadareja and Bronka Rzepkowski.

The team has chosen the name of Marmotte[1] after discovering on the net, that marmots are able to foresee the return of spring, 48 hours before official weather forecasts. However, if the team managed to impose this name and convince both institutions that this was not an indication of the pace they intended to work at, they never succeeded in persuading Cepii nor Cepremap to

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[*] CEPII.

[♦] CEPREMAP

[1] The acronym of Marmotte has a clear meaning, at least in French: Modèle à Anticipations Rationnelles Multinational Optimisant la Théorie et les Techniques Econométriques.

sponsor a marmot in North Canada. All the economists who have worked on the model are sorry to have missed the opportunity to help a species threatened with extinction.

A feasibility study of Marmotte was launched in 1998, during which the Marmotte team benefited from advice and support by André Dramais from the European Commission. He gave the team access to the Quest 2 model of the European Commission, which has been simulated at Cepii. This was an invaluable help. The team also worked with Multimod Mark 3, and did numerous simulations with it. Michel Juillard, Douglas Laxton and Pierre Malgrange gave useful advice at this stage of the research. The results of the feasibility study were presented to the Cepii council on 16 December 1998. The council agreed that Cepii should build Marmotte over the years 1999-2000. 2001 should be used to improve specific features of the model and to evaluate its properties.

At the beginning of 1999, a scientific committee was constituted to evaluate the work of the Marmotte team and to give it advice on the orientation of its research. The team included Jean Cordier and Pierre Sicsic from the Bank of France, Michèle Debonneuil, Paul Zagamé and Reza Lahidji of the Commissariat Général du Plan, André Dramais and Werner Roeger of the European Commission, Guy de Monchy of the Direction de la Prévision, Eric Dubois of the Ministry of Social Protection, Pierre-Yves Hénin and Pierre Malgrange of Cepremap, Gérard Maarek of the Caisse Nationale du Crédit Agricole, Laurent Bouscharain and Françoise Maurel of the Insee. The scientific contribution of this committee was considerable. The most original feature of Marmotte, its putty clay production technology, was a suggestion of the committee, which was enthusiastically accepted by the team.

The Marmotte team is grateful to Agnès Bénassy, a former deputy-Director of Cepii, for fruitful discussions. Agnès had the strong intuition and feeling required to quickly discover the defects of the model. However, she was too honest to use these qualities to invent a story explaining why these defects were perfectly normal and should not require any correction. The new Director and deputy-Director of Cepii, Lionel Fontagné and Florence Legros were very supportive of the Marmotte project, and they played an extraordinary role in the diffusion of the model in foreign countries and international organisations.

At the beginning of July 2001, Cepii organised an Enepri[2] meeting at Royaumont Abbey on the comparison of the results given by multinational models. The most prestigious models were represented at this meeting, and the first official presentation of Marmotte was given before its elders. Florence Legros and Bronka Rzepkowski played a fantastic role in the organisation of the meeting (and they selected the site, which is wonderful at the beginning of summer). Michel Juilliard made a synthesis of the results of the simulation of the models and compared them. All the participants of the meetings were very encouraging with the new-born Marmotte. This meeting was very precious for the Marmotte team, which could thus improve some features of their model. The assumption of sticky prices, required to obtain short run Keynesian properties of the model, was introduced after this meeting.

Marmotte is managed and simulated under Troll. The Troll team was very helpful, Tanguy Deschuyteneer and Peter Hollinger answered very quickly all the questions of the Marmotte team and gave valuable advice which facilitated very much the development of Marmotte.

[2] European Network of Economic Policy Research Institutes.

I INTRODUCTION

Marmotte is a multinational model focusing on the medium-term, built by the Cepii in co-operation with the Cepremap. It includes 17 industrialised countries: the 14 members of the European Union (Luxembourg and Belgium are merged), the United States, Japan and Canada. Marmotte provides both a careful description of these economies, including significant structural asymmetries, and a comprehensive description of interdependence, through the trade and capital flows. It is thus an appropriate tool to investigate the international transmission of shocks of a different nature.

The reasons underlying the construction of a new multinational model and its general philosophy are presented in Section 1. Strategic methodological choices in the building of Marmotte have followed some guidelines that are exposed in Section 2. The main economic specifications of the model are described in Section 3. Lastly, the plan of the monograph is given in Section 4.

I.1 Why a new model?

Before engaging in the construction of Marmotte, Cepii and Cepremap analysed two existing models that seemed particularly interesting with regard to their goals: Multimod Mark 3 built by the IMF and Quest 2 built by the European Commission. Multimod is a yearly model of the G7 countries, with limited details of national accounts. Quest 2 is a quarterly model, including each member of the EU, and presenting more detail for countries and agents' accounts. These two models have strong theoretical contents. Agents optimise inter temporally and form rational expectations. The advantage of such modelling is to link the short run (characterised by sticky prices and nominal wages) and the long run (which is strongly supply determined) in a convincing way. Forward expectations induce an influence of the long run on the short run equilibrium, and shorten the lag required before supply constraints act on the dynamics of the economy. Moreover, forward structural models have to comply with some important consistency conditions before they can be simulated. They require more discipline than traditional backward models that can be simulated even when they are unstable and when their computed path diverges in the long run to meaningless values. Besides, structural rational expectations models are consistent with the most recent developments in open macroeconomics. Finally, a model with a rigorous theoretical content is also much easier to understand, use and control. Clear interpretations of the results can be given, and discussions with economics experts of theoretical modelling or of business conditions in the world economy are much easier.

Cepii and Cepremap were quite impressed by the qualities of Multimod Mark 3 and Quest 2. However, they decided to build a new multinational model, and this for several reasons. First, Multimod Mark 3 is limited to G7 countries and Cepii and Cepremap wanted to have a model including every member of the EU. Secondly, Quest 2 requires many data, with quarterly periodicity that are especially built for the model, by economists at the European Commission, and that are not available in the public domain. The Marmotte team wanted to use only easily available and well-documented statistical data, such as for instance the yearly national accounts of the Oecd, which benefit from some homogenisation by the Oecd's statisticians. This requirement was necessary to give the users of the model the possibility of easily modifying some features of Marmotte for specific purposes and of adding data to the databank of the model. An annual frequency should also be enough for a medium term tool.

Another reason for building a new model was that Cepii and Cepremap wanted to make specification choices different from those of Multimod and Quest. Multimod does not include a labour market, nor unemployment. Inflation is explained by a Phillips curve, in which unemployment is substituted by the output gap. Cepii and Cepremap wanted to have a full specification of the labour market, with a wage curve (the pseudo-supply of labour) instead of a Phillips' curve.

The consumption function in Multimod and in Quest is based on the assumption of finite-lived households, with an uncertain date of death. This specification, developed by Blanchard, is very elegant and popular in computable general equilibrium models. However, if its qualitative properties are attractive, its quantitative properties are more questionable, and there are no econometric works validating this specification. On the contrary, numerous econometric works are based on well-documented puzzles of consumption behaviour, for instance the inertia of consumption faced with permanent shocks. Thus, the Marmotte team, taking inspiration from empirical studies, preferred keeping the assumption of infinitely lived households, but with non-separable utility functions taking account of habits' formation.

Multimod and Quest chose a putty-putty technology, and an investment equation based on Tobin's *q*. On this assumption, the capital intensity of the production process can be changed instantaneously and without cost. Thus, in a competitive framework, the production factors fully and instantaneously adjust to current economic conditions. This means that "realistic" changes in real wages or in the capital costs lead to very significant and quick moves in the demand for labour and capital.

However, actual employment and capital stock exhibit much weaker movements than those predicted above. One way to decrease the cost-sensitivity of production factors consists in assuming non-linear adjustment costs (usually quadratic costs). This results in smoother dynamic adjustments of labour and capital. However, this specification rests upon an *ad hoc* assumption without wholly rigorous micro economic foundations and empirical verification.

So, it was decided to assume a putty clay technology. Such technology makes it easier to stabilise the model compared to a putty putty technology. It furthermore provides a convincing explanation of medium-run movements in the distribution of income between wages and profits.

I.2 Strategic methodological choices in Marmotte

The basic aim the Marmotte team tried to reach was to build a user-friendly model, which economists could easily learn to simulate. Changing its equations, adding countries or building a scenario should also be a simple task. For this purpose the theoretical background of the equations had to be very well-known and acknowledged by professional economists. All the data should come from international institutions' databanks and be available under Wefa, so that they can automatically be managed under Aremos[3]. It was also decided that all the econometric estimations should be made using Tsp and the simulations of Marmotte using portable Troll. The advantage of these tools is that they are widely available and of renowned quality.

[3] Aremos is an integrated system designed for data base management and manipulation.

General features of Marmotte

Marmotte includes 17 industrialised countries: the 14 members of the European Union (Luxembourg and Belgium are merged), the United States, Japan and Canada. Each country is modelled by the same system of about 50 equations. The Marmotte team did not see any reason to assume different specifications for different industrialised countries. Moreover, by assuming the same specification for each country, the equations of Marmotte can be written for only one country: a device of Troll (called generic) can automatically write the equations of all the other countries. Thus, the users of Troll have to check only for the syntax and the consistency of 50 equations and not of 50*17 equations[4]. 50 equations seemed a reasonable size, the same as for Multimod and Quest models. The values of the parameters of the equations may differ between countries: many of them were estimated by econometric methods, others were calibrated (especially the supply-side equations). More precisely Marmotte identifies four kinds of differences between countries.

1) countries differ in size and in the various ratios between national aggregates;
2) countries differ in their openness and in the geographic structure of their bilateral trade flows;
3) countries differ in their exchange rate regimes;
4) countries differ in values taken by the different behavioural and structural parameters.

The three first differences are well measured. But the evaluation of the last one rests on econometric estimations based on macroeconomic data. These last parameters account for the structural asymmetries across countries. In order to make the simulations of MARMOTTE easier to interpret and discuss with experts of business conditions in Europe, two versions of the model were built. In the first one, all the countries have the same behavioral and structural parameters and in the second one, the value of these parameters are allowed to differ between countries.

Database, estimation and simulation

The model has an annual frequency, which is well suited for medium-term forecasts. Besides, the construction and the management of an annual model are easier than that of a quarterly one. The annual data are also more reliable and they are often harmonised by international institutions. Thus, they can easily be managed using an automatic program. For Marmotte, all the data are taken from Wefa and are automatically extracted and transformed with Aremos.

Panel data techniques were used to estimate the econometric relationships. Each behavioural equation was estimated independently for the 17 countries. The error terms of the various countries were assumed to be correlated, and their covariance matrix was approximated by using factor analysis techniques. As each equation includes several endogenous variables, and

[4] This goal was not wholly achieved. First, a few equations are specific to some countries, for instance the reaction policy of the ECB. Second, the numbers of lags and leads in the putty clay specifications differ across countries, and it was impossible to set parameters for this length. So a few equations had to be explicitly written for each country .

sometimes expected variables, the estimation was based on GMM. A strategy of nested tests was used to decide which parameters could be assumed to differ or not across countries.
A baseline account of the model was built for the period 2000-2005. This account is based on forecasts made by Oecd. The values taken by lagged variables before 2000 were entered in the databank. The world economy was assumed to follow a reasonable balanced growth after 2060. The baseline account was extended after 2005 to converge to this balanced growth path along a smooth path. Of course, the baseline account satisfies all the accounting identities. To make it also satisfy the other equations of the model under the assumption of perfect forecast, additive or multiplicative residuals were added to these equations. Thus, the model is trivially consistent with its baseline account. Then, it can be simulated in the neighbourhood of this baseline account, after having shocked some of its exogenous variables. Simulations are computed with the Troll software.

I.3 Main economic specifications of Marmotte

The most original feature of Marmotte rests on the supply side, which is modelled by a putty-clay technology: capital can be substituted by labour only in the long run. In each period, a new vintage of capital is installed, whose capital-labour ratio chosen in the menu of technology available remains unchanged until its scrapping. The capital intensity and the expected lifetime of the new production unit result from an inter-temporal optimisation of its expected discounted profitability.

This technology is especially well suited to the analysis of medium term movements in the allocation of national income between wages and profits. It can explain the stickiness of employment and investment and the way a change in the structure of the labour market will progressively change production technology and the working of the economy. According to this specification, output can increase because of a rise in the age of the oldest production units kept in activity, and because of *past* investments.

Another specific feature of Marmotte is that households optimise inter-temporally with a non-separable utility function, which takes into account habits' formation. This induces some stickiness in consumption behaviour.

Other features of the model (imports, exports, wages and prices equations with some stickiness of nominal values, interest rate parities, monetary rules, etc.) are quite classical. In the wage curve (the pseudo-supply of labour), the real cost of labour decreases with the inflation rate (the stickiness of the nominal wage rate), and with the ratio between the production and consumption price, while it increases with the rate of employment and the current productivity of labour.

The model assumes inter-temporal equilibrium for both the budgets of governments and the balance of payments of nations. To stabilise the model, the uncovered interest rate parity has been amended by adding a risk premium that will "punish" (or "reward") a country for its greater (or lesser) impatience, and so for its tendency to over-borrow (over-lend). This premium is related to the external asset position of the country. If the external indebtedness of a country increases, a higher risk premium will be attached to the currency of that country. In the dynamic model as the exchange rate affects the import and export equations, any increase in external indebtedness will have a depressing effect on imports and the opposite effect on exports.

Monetary policy is implemented by the central bank of each non-EMU country and by the European Central Bank for the Euro zone countries. The central banks' goal is to stabilise inflation around a target according to a Taylor type monetary rule.[5]

Finally, Marmotte assumes some stickiness of production prices. In each country, the rate of variation of this price is proportional to the logarithm of the ratio between the demand and the supply of goods produced by the national private sector. Current production is assumed to be equal to demand. Supply determines potential production. In the long run, effective and potential outputs are equal. This stickiness of prices was introduced in Marmotte to provide some Keynesian features in the short run.

I.4 Plan of the monograph

The **first chapter** of the monograph presents the modelling of the production of goods and of the demand of production factors. Whereas macro econometric models usually assume a putty-putty technology, we would expect the current technology menu to be only available to the newly-created units of production. This is precisely what the putty-clay specification does. In this framework, current economic conditions affect the capital intensity of new production units (their technological choice) and the number of units built (investment in the economy). Since this capital intensity cannot change in the future, the expected lifetime of the new production units is part of investment decision-making. Both capital intensity and expected lifetime are set to maximise the expected discounted cash flows minus installation costs. Older production units keep the technology they were given at their creation. However, current economic conditions affect their profitability and lead to the scrapping of non-profitable units. Hence, the aggregate capital-labour ratio changes gradually with the flows of investment and the scrapping of old obsolete production units. The decisions of firms can be aggregated to give the national investment, employment and production levels of the current period. Putty-clay investment may thus provide medium-term dynamics in the distribution of income.

This specification has some other advantages. The irreversibility of investment is embedded in the model and firing costs can easily be introduced. This provides a convincing foundation to the stickiness of employment.

The **second chapter** describes the modelling of the consumption behaviour of households. Macro-econometric studies on consumption often rely on extensions of the permanent income model, in which a representative consumer maximises the discounted sum of his instantaneous utilities. Taking his preferences into account, the representative agent chooses between consuming today and saving to consume later by comparing the effects of each of these two choices on his welfare.

The most popular version of this model assumes a time-separable utility function for households: their welfare in a given period depends only on their consumption in this period, independently of past consumption. However, this specification seems to underestimate both

[5] The fact that central banks use Taylor rules, where the nominal interest rate is related to the inflation rate, introduces hysteresis into the model. The nominal anchors, which determine the output deflators in the various countries, are prices inherited from the past~~The fact that Central Banks use Taylor's rules, where the nominal interest rate is related to the inflation rate, introduces hysteresis into the model. The nominal anchors, which determine the output deflators in the various countries are prices inherited from the past~~.

the inertia of consumption relative to permanent income and the sensitivity of consumption to current income. To account for the excess smoothness of consumption, Marmotte reconsiders the assumption of a time-separable utility function, to take into account habit formation. It derives an arbitrage condition from an iso-elastic instantaneous utility function with current and past consumption as arguments. The lack of sensitivity to current income can be related to the existence of liquidity constraints that undermine the model's assumption of competitive financial markets. In spite of the financial deregulation implemented in the past two decades, part of households may still be unable to borrow against their future income. This liquidity constraint is difficult to integrate in a tractable way. In particular it introduces strong non-linearity in the optimisation programme. A convenient solution consists in assuming two types of households whose proportion in the economy is constant over time. The households of the first group are liquidity-constrained. Although they want to borrow, they find no counterpart in the financial market. This means that they consume all their current income. The households of the second group have free access to financial markets and behave according to the arbitrage equation.

This chapter first presents the theoretical framework of the behavioural equations retained for Marmotte. Then, it provides the estimation of the parameters and discusses the relevance of country-specific values for the 17 countries of the model.

Chapter three successively presents the accounting identities, the wage curve (the pseudo-supply of labour), the foreign trade equations and the monetary and exchange rate parts of the model.

In Marmotte, each country produces one specific good, imperfectly substitutable for the goods produced by its partners. However, for international trade there is a distinction between primary commodities (including oil) on the one hand, and manufactured goods and services on the other hand. Each country uses a composite good, which is the same for consumption and investment. There are three agents per country: the Government, private firms and households. A first series of equations deals with the equilibrium of goods and services. They include an equation linking the inflation rate to the desequilibrium between potential production and effective demand. This equation introduces price stickiness, which is necessary to get Keynesian properties in the short run. A second series of equations relates the various prices prevailing in a country to the production price and foreign prices. Another series of equations defines the budget deficit of the Government, and relates it to the accumulation of public debt. The divergence of this debt is prevented by an automatic stabiliser, linking lump-sum taxes and transfers to the ratio of public debt to GDP. A last block of equations defines the trade balance and relates its deficit to the dynamics of foreign indebtedness.

The second section of Chapter 3 presents a simple theoretical formalisation of wage setting, based on a wage curve in which labour cost depends on the employment rate, labour productivity, prices and the wedge between real labour cost for firms and the purchasing power of nominal wages for wage earners. It also introduces nominal rigidities of wages based on overlapping contracts with unequal length. Then, it gives an econometric estimation of private wage behaviour on the panel of the 17 industrialised countries of Marmotte with yearly data. Panel estimation allows identifying deep structural differences between countries. This is particularly important as industrialised countries' labour markets display great heterogeneity concerning wage bargaining processes, degrees of job protection, and the provision of replacement incomes, etc.

The third section of Chapter 3 presents the specification and the estimation of the foreign trade block of Marmotte. The specification of the equations of this block follows a well-established tradition. These equations have an error correction structure. In the long run the quantities of exports and imports depend on demand and price competitiveness. Import and export prices depend on national production prices and on the prices set by foreign competitors.

The definitions and computations of foreign demand indicators and price competitiveness are an especially important part of this section. They are based on the structure of bilateral trade flows, which are an essential component of the structural asymmetries between countries. The econometrics of this section is based on the panel data of the 17 countries of Marmotte.

The fourth section of Chapter 3 presents the monetary and exchange rate part of the model. Each country has two interest rates, related by a term structure equation. The short term interest rate of each country is related to the interest rate in the US, by a parity relation. It includes a risk premium, which increases with foreign indebtedness. The central bank of each country (and the ECB for the euro zone) uses a Taylor rule.

Chapter 4 presents the simulation methodology of the model. Blanchard and Kahn (1980) established conditions for the existence and uniqueness of a solution for rational expectation models. However, these conditions only apply to linear models whose coefficients do not depend on time, and such that the exogenous variables can be assumed to be constant after some time. Marmotte has very different features. However, it can generate a balanced growth path. Then, the linear approximation of the model around its balanced growth path can be computed, and the solution may be required to converge to this path when time increases indefinitely. As some of the coefficients of the linear approximation appear as geometric functions of time, the results by Blanchard and Kahn can not be directly used, unless all the variables are put on a common trend. In a first step, the results by Blanchard and Kahn are presented with the extension to the case of hysteresis, developed by Giavazzi and Wyplosz (1986). Afterwards, sufficient local conditions of existence and uniqueness, which can be applied to Marmotte, are established.

The second section of Chapter 4 gives the details of the application of the above methodology to Marmotte, and explains the technique used to simulate the model.

Chapter 5 presents some basic simulations of Marmotte.

In Europe, the autonomy and the co-ordination of national fiscal policies, the roles of a common monetary policy and of the exchange rate system implied by EMU have important consequences on the international transmission of economic shocks. In a monetary union, symmetric shocks may induce asymmetric effects on countries displaying significant structural asymmetries. A common monetary policy alone will not be able to absorb the resulting cyclical differences.

The main purpose of this chapter is to assess the differentiated responses of European countries to standard shocks: demand and supply shocks. The demand shock is simulated through a permanent increase in government expenditures, by 1% of the GDP. The supply shock corresponds to a productivity shock that rises permanently output by 1%, in the long run. Furthermore, the effects of these shocks on Europe are investigated with respect to their symmetric or asymmetric nature, i.e. by distinguishing their origin. A shock occurring in the United States stands for a symmetric shock on Europe, whereas if it occurs in a country of the euro area, such as Germany, it represents an asymmetric shock. Four simulations are thus computed. The effects of these shocks on the US and Germany (the countries where shocks occur), France and the UK (where monetary policy is defined at a national level) are given and commented.

Appendix 1 presents the econometric methodology of Marmotte. Many behavioural equations of Marmotte were estimated for a panel of industrialised countries. Estimating a macroeconomic equation for a panel of countries has become popular. According to this approach, the values of the parameters of this equation may differ between countries, but not its specification. Using a panel estimation helps to get more robust and precise empirical findings: as these countries share some common structural features, each country's estimation benefits from information brought by its partners. Moreover, panel estimation allows deep structural differences between countries to be identified.

As the error terms of the various countries are probably correlated, and as the structure of these correlations is probably more complex than the one allowed by error component models, SUR appear to be the most natural way to carry out this estimation. However, the presence of endogenous and anticipated explanatory variables requires the use of instrumental variables and GMM, instead of generalised least squares. Moreover, to get a robust estimation of the covariance matrix of error terms, recent developments of factor analysis were used. Finally, a strategy of tests, going from general to specific, to determine which parameters change between countries and which parameters take the same value among all countries, is presented.

Appendix 2 gives the equations of Marmotte as they appear in the Troll programme. Both the steady state equations and those corresponding to the dynamic version of the model are provided. The list and the value of the parameters are also given.

Appendix 3 describes the construction of the database needed to run Marmotte. It starts with a database filled with observed data. This initial database is extracted from Oecd data with an Aremos program. The initial database spreads from 1970 to 1996 (it can be updated automatically). Some data are not available for this period. With this basic database, the final aim was to extrapolate it with Oecd medium-term forecasts (until 2005) and, second, to complete it to get a full database, useful for the model, that is ranging from 1910 to 2300. This second step provides the baseline for Marmotte. The last step before the simulation of the model concerns the calibration of the supply-side of the model, and the computation of the residuals.

CHAPTER I

PRODUCTION OF GOODS AND DEMAND OF PRODUCTION FACTORS[6]

Macroeconometric models usually assume a putty-putty technology: the capital intensity of the production process can be changed instantaneously and without cost. Thus, in a competitive framework, the factors of production fully and instantaneously adjust to current economic conditions. This means that “realistic” changes in real wages or in the cost of capital lead to very significant and quick moves in demand for labour and capital. Moreover, the quick adjustment of the capital stock should cause huge variations in the flows of investment.

However, actual employment and capital stock exhibit much weaker movements than those predicted above. Hence, the integration of this theoretical framework in a realistic model requires some improvements. One way to decrease the cost-sensitivity of production factors consists in assuming nonlinear adjustment costs (usually quadratic costs). This results in smoother dynamic adjustments of labour and capital. However, this specification rests upon an *ad hoc* assumption without wholly rigorous microeconomic foundations and empirical verification. Moreover, it is not a fully convincing way to model the irreversibility of investment and the firing costs of labour. Finally, the putty-putty framework is unable to give simple, acceptable explanations for the medium term movements in the wage share in value added, which are observed in some European countries (see for instance Blanchard, 1997, Prigent, 1999). Although adjustment costs smooth the dynamics of factor demands in the short run, they are far from sufficient to produce medium term changes in the income distribution between capital and labour.

A key feature of the putty-putty specification, that is central to its empirical failure, is that all the vintages of capital have the same capital intensity. On the contrary, we would expect the current technology menu to be only available to the newly created units of production. This is precisely what the putty-clay specification does. In this framework, current economic conditions affect the capital intensity of the new production units (their technological choice) and the number of these units created (investment in the economy). The other production units keep the technology they were given at their creation. However, current economic conditions affect their profitability and lead to the scrapping of non-profitable units. Hence, the aggregate capital-labour ratio changes gradually with the flows of investment and the scrapping of old obsolete production units. Putty-clay investment may thus provide medium term dynamics in the distribution of income.

This specification has some other advantages. The irreversibility of investment is embedded in the model and firing costs can easily be introduced. This gives a convincing foundation to the stickiness of employment.

Despite all its advantages, the putty-clay technology suffers from a serious drawback. Its implementation in a macroeconomic model is cumbersome for two reasons. First, the model has a long memory since it keeps track of all the vintages of capital created in the past, that are still in working order. Thus, the model has “variables with long lags”. Second, the planning horizon of investors stretches far into the future. More precisely, the decision concerning the new

[6] This chapter is based on Cadiou, Dées and Laffargue (2001).

production units involves forward variables that cover the expected lifetime of these units. The model has then “variables with long leads”.

However, these problems can be easily overcome nowadays. Models with variables presenting long leads and lags can be solved with powerful algorithms (for instance those implemented in Troll), and simulation time is decreasing with the improvement of personal computers.

In this chapter we will introduce the specification of the technology of firms and we will determine their decisions. At each date, firms build a number of new production units, which will start to produce one period later. They have to choose the capital intensity embodied in these units. Since this capital intensity cannot change in the future, the expected lifetime of the new production units is part of the decision-making. Both capital intensity and expected lifetime are set to maximise the expected discounted cash flows minus installation costs. Moreover, firms reassess the profit that each older unit would make during the current period if it were kept into activity. When this profit is negative the unit is scrapped. Then, we can aggregate the decisions of firms and get the investment, the employment and the production of the current period.

I TECHNOLOGY AND INVESTMENT COST

We consider a representative firm[7], on a perfectly competitive goods market, which must make choices at period t. At this time, the firm decides to acquire k_t new units of capital. It also chooses the technology embodied in this capital. The technology menu is characterised by an *ex ante* first-order homogenous production function:

$$F(k_t, A_t(1+g)^t n_t) = z[ak_t^{1-1/s} + (1-a)(A_t(1+g)^t n_t)^{1-1/s}]^{s/(s-1)},$$ with: $z, s > 0, 0 < a < 1$.

This equation determines the amount of goods produced by combining k_t units of capital in equipment with n_t units of labour. $A_t(1+g)^t$ represents the efficiency of technology available at time t. Efficiency is divided between two factors: the first can be exogeneously changed in simulations and has no trend, the second is a constant positive trend. s is the *ex ante* elasticity of factor substitution. The capital intensity $\boldsymbol{k}_t$ chosen at time t for the entire lifetime of the capital units is defined by:

$$\boldsymbol{k}_t = k_t / (A_t(1+g)^t n_t).$$

The production function can be rewritten as:

(1) $$F(\boldsymbol{k}_t, 1) = z[a\boldsymbol{k}_t^{1-1/s} + (1-a)]^{s/(s-1)}$$

[7] As firms are identical, the choices made by any individual firm also hold at the aggregate level.

In the future, the vintage of capital created at t will either be used with capital intensity k_t or be scrapped.

We define one unit of production created in period t as the combination of one unit of labour and $k_t A_t (1+g)^t$ units of capital. This unit of production produces $A_t (1+g)^t F(k_t, 1)$ units of goods. During period t, n_t new units of production are created. At the end of each period, a fraction d of the units suffer irreparable failures and stop producing. d can be seen as the rate at which production units disappear for all reasons but macroeconomic conditions (bad management, mistakes or technical difficulties in the implementation of production). In that sense, d is equivalent to an exogenous depreciation rate.

For every unit, production starts in the period following its installation. The units created during period t are productive from period $t+1$.

Aggregate investment in equipment at time t is defined by:

(2) $$I_t = A_t (1+g)^t k_t n_t$$

The cost of one unit of capital, including installation costs, expressed in units of good produced, is exogenous and equal to $c_t > 0$.

II VALUE OF A NEW PRODUCTION UNIT

We define r_t as the nominal interest rate of an asset with a maturity of one period, available at time t. We also define $w_t (1+g)^t$ as the nominal wage paid to each worker of the production unit for the work done during period t (thus w_t can be interpreted as wage per unit of efficiency). Let us consider a unit of production created at period t. When capital is scrapped, at period $t+T(t)$, firing the workers costs $p_{t+T(t)} x^f_{t+T(t)} (1+g)^{t+T(t)}$ where $p_{t+T(t)}$ is the price level at time $t+T(t)$ and $x^f_{t+T(t)} (1+g)^{t+T(t)}$ is the real firing cost[8]. If the production unit suffers an irreparable failure, however, no firing cost is incurred.

We define by $V_{s,t}$ the present value of the future cash flows of the production unit built at t, measured at period s, i.e. the value of the production unit for the units that have not failed until that time. In case of failure, this value is zero.

We have the following arbitrage condition:

8 This means that the unit of production has been active from period $t+1$ to period $t+T(t)$, that is for $T(t)-1$ periods. We have assumed that a firm which closes a production unit supports firing costs on the total of labour which was employed on this unit. We could have assumed instead that the creation of new production units at period t enables firms to save a fraction of the firing costs incurred by scrapping units at the next period, when the new unit becomes active. Labour could simply be displaced from the closed unit to the new unit.

$$(1-d)V_{s+1,t} + (1-tr_{s+1})\,p_{s+1}A_t(1+g)^t F(k_t,1) - w_{s+1}(1+g)^{s+1}A_{s+1} = (1+r_s)V_{s,t}$$

where tr is the tax rate on production.

In period s, the owner of a production unit can either sell it and invest the proceeds in the financial markets or hold it. In the first case, it will get $(1+r_s)V_{s,t}$ in period *s+1*. In the second case, the unit will yield an after tax profit in period *s+1* equal to: $(1-tr_{s+1})\,p_{s+1}A_t(1+g)^t F(k_t,1) - w_{s+1}(1+g)^{s+1}$. Besides, in period *s+1*, the unit will either go bankrupt, with probability d, or still be in working order, with probability ($1-d$).

Now, we must face a technical difficulty. The model is written and simulated in discrete time. All the units of the same vintage are identical. Thus, in a given period, all the units of a given vintage either make a non negative profit and must be kept in activity, or make a positive loss and must be wholly scrapped. Thus, supply available for a given period can only take a finite number of values, equal to the age of the oldest available vintage. This feature of the model is unconvincing[9]. We cannot correct it by assuming that part of a given vintage is scrapped at the beginning of a period, and the rest scrapped at the end of the same period: as all the units of a given vintage are identical, they all win money, or lose money in a given period and they all must be scrapped at the same time.

To solve this difficulty we will assume that a vintage can be scrapped at an intermediary time inside a period. More precisely, production made during a period is available, sold and used during this period. There is a unique price and a unique condition of equilibrium on the market of goods for the whole period. However, we will assume that the wage curve, which will be introduced later, define the average wage rate over the current period. Inside the period firms will pay a sequence of instantaneous wage rates which grow at rate g. Similarly we will assume that the firing cost which were defined above apply when the production unit closes at the end of the period. If the unit is scrapped before, firing cost are reduced by a discount factor equal to g.

In the model, payments are made each period. Thus, the manager of a production unit has an incentive to postpone the closure of its unit until the beginning of next period (for instance from November the 1st to January the 1st), to benefit from the possibility to postpone firing costs for a whole period and earn interest on this amount of money. To remove this unreasonable feature of the model, we will assume that when a unit is scrapped inside a period, firing costs are allocated between the current and the previous periods.

Let us be more precise. The expected scrapping date of a new unit built at period t is $t+T(t)$, which does not necessarily represent an integer number of years. We define $\overline{T}(t)$ as the

[9] The consequences of this feature are more serious when the model is written on a yearly basis, like ours, than on a quarterly basis. But even in this last case the discontinuities in the supply of goods to the economy would give to the response of the model to shocks a lack of smoothness which would be difficult to accept.

integer part of $T(t)$, and $\Delta T(t)$ as its decimal part[10]. Wages and firing costs at time $t+T(t)$ are: $w_{t+\overline{T}(t)+1}(1+g)^{t+\overline{T}(t)+\Delta T(t)}$ and $x^f_{t+\overline{T}(t)+1}(1+g)^{t+\overline{T}(t)+\Delta T(t)}$.

We now have the terminal arbitrage condition:

$$
\begin{aligned}
&(1+r_{t+\overline{T}(t)})V_{t+T(t),t} = \\
&+\Delta T(t)\Big[(1-tr_{t+\overline{T}(t)+1})\,p_{t+\overline{T}(t)+1}A_t(1+g)^t F(\boldsymbol{k}_t,1) - w_{t+\overline{T}(t)+1}(1+g)^{t+\overline{T}(t)+\Delta T(t)}\Big] \\
&- p_{t+\overline{T}(t)+1}x^f_{t+\overline{T}(t)+1}(1+g)^{t+\overline{T}(t)+\Delta T(t)}\left(\frac{p_{t+\overline{T}(t)+1}}{p_{t+\overline{T}(t)}}\frac{1-\boldsymbol{d}}{1+r_{t+\overline{T}(t)}}\right)^{\Delta T(t)-1}\Bigg\}
\end{aligned}
$$

The value of a new production unit is obtained by summing the arbitrage equations until the scrapping date:

$$
\begin{aligned}
V_{t,t} = &\sum_{s=t+1}^{t+\overline{T}(t)}\Big[(1-tr_s)p_sA_t(1+g)^tF(\boldsymbol{k}_t,1) - w_s(1+g)^s\Big](1-\boldsymbol{d})^{s-t-1}\Big/\left(\prod_{t=t}^{s-1}(1+r_t)\right) \\
&+ p_{t+\overline{T}(t)+1}\Big\{\Delta T(t)\Big[(1-tr_{t+\overline{T}(t)+1})A_t(1+g)^tF(\boldsymbol{k}_t,1) - \big(w_{t+\overline{T}(t)+1}/p_{t+\overline{T}(t)+1}\big)(1+g)^{t+\overline{T}(t)+\Delta T(t)}\Big] \\
&- x^f_{t+\overline{T}(t)+1}(1+g)^{t+\overline{T}(t)+\Delta T(t)}\left(\frac{p_{t+\overline{T}(t)+1}}{p_{t+\overline{T}(t)}}\frac{1-\boldsymbol{d}}{1+r_{t+\overline{T}(t)}}\right)^{\Delta T(t)-1}\Bigg\}(1-\boldsymbol{d})^{\overline{T}(t)}\Big/\left(\prod_{t=t}^{t+\overline{T}(t)}(1+r_t)\right)
\end{aligned}
$$

III CHARACTERISTICS OF THE NEW PRODUCTION UNITS

The investor which decides to build a new production unit pays its building cost during installation period t: $p_tc_tA_t(1+g)^t\boldsymbol{k}_t$. It is aware that this unit has a probability $\boldsymbol{d}$ to go bankrupt in the same period, and a probability ($1-\boldsymbol{d}$) to remain in working order. Its value at the end of period t, is then: $(1-\boldsymbol{d})V_{t,t}$. The investor determines the planned lifetime and the capital intensity of the unit by maximising the difference between its value and its cost:

$$
\Psi_t\big(\overline{T}(t),\Delta T(t),\boldsymbol{k}_t\big) = (1-\boldsymbol{d})V_{t,t} - p_tc_t(1+g)^tA_t\boldsymbol{k}_t.
$$

<u>Expected life time</u>

The integer part of the expected lifetime, $T(t)$, is defined by:

$$
\text{(3)}\qquad
\begin{cases}
\Psi_t\big(\overline{T}(t),\Delta T(t),\boldsymbol{k}_t\big) - \Psi_t\big(\overline{T}(t)-1,\Delta T(t),\boldsymbol{k}_t\big) > 0 \\
\Psi_t\big(\overline{T}(t)+1,\Delta T(t),\boldsymbol{k}_t\big) - \Psi_t\big(\overline{T}(t),\Delta T(t),\boldsymbol{k}_t\big) < 0
\end{cases}
$$

---

[10] This notation results from the fact that period $\overline{T}(t)+1$ starts at time $\overline{T}(t)$ and ends at time $\overline{T}(t)+1$.

The decimal part of the expected scrapping date of the unit $\Delta T(t)$ is determined by the following first-order condition:

$$\frac{\partial \Psi_t(\bar{T}(t), \Delta T(t), \boldsymbol{k}_t)}{\partial \Delta T(t)} = 0$$

Thus:

$$(4)\qquad \left((1 - tr_{t+\bar{T}(t)+1})(1+\boldsymbol{g})^{-\bar{T}(t)-\Delta T(t)} A_t F(\boldsymbol{k}_t, 1) - \left(\frac{w_{t+\bar{T}(t)+1}}{p_{t+\bar{T}(t)+1}} \right) \right) - \Delta T(t) \left(\frac{w_{t+\bar{T}(t)+1}}{p_{t+\bar{T}(t)+1}} \right) \ln(1+\boldsymbol{g})$$
$$- x^f_{t+\bar{T}(t)+1} \left(\frac{p_{t+\bar{T}(t)+1}}{p_{t+\bar{T}(t)}} \frac{1-\boldsymbol{d}}{1+r_{t+\bar{T}(t)}} \right)^{\Delta T(t)-1} \left(\ln(1+\boldsymbol{g}) + \ln\left(\frac{p_{t+\bar{T}(t)+1}}{p_{t+\bar{T}(t)}} \frac{1-\boldsymbol{d}}{1+r_{t+\bar{T}(t)}} \right) \right) = 0$$

We can show:

Proposition 1: In the neighbourhood of the steady state and for small growth rates, interest rates and depreciation rates, equation (4) determines a unique integer value for $\bar{T}(t)$ and a unique real value included between 0 and 1 for $\Delta T(t)$. These values satisfy inequations (3)
Proof: See Appendix 1. The appendix includes considerations on practical ways to simulate the model.

<u>Capital intensity</u>

The capital intensity chosen by investors for the new production units is given by the first-order condition: $\dfrac{\partial \Psi_t(\bar{T}(t), \Delta T(t), \boldsymbol{k}_t)}{\partial \boldsymbol{k}_t} = 0$.

$$(5)\qquad F_1^{'}(k_t, 1) \left\{ \sum_{s=t+1}^{t+\bar{T}(t)} p_s (1 - tr_s)(1-d)^{s-t-1} / \prod_{t=t_0}^{s-1} (1 + r_t) \right.$$
$$\left. + \Delta T(t) p_{t+\bar{T}(t)+1} (1 - tr_{t+\bar{T}(t)+1})(1-d)^{\bar{T}(t)} / \prod_{t=t}^{t+\bar{T}(t)} (1 + r_t) \right\} = p_t c_t / (1-d)$$

where $F_1^{'}(\boldsymbol{k},1)$ is the marginal productivity of capital.

We will now assume free entry of firms, which means that the financial value of a new production unit is equal to its building cost (taking into account the possibility of bankruptcy):

$$(1-\boldsymbol{d})V_{t,t} = p_t c_t A_t (1+\boldsymbol{g})^t \boldsymbol{k}_t$$

By combining the last two equations we get (proof available on request):

$$\begin{aligned}
&\sum_{s=t+1}^{t+\overline{T}(t)}\left[(1-tr_s)\,p_s A_t F_2'(\boldsymbol{k}_t,1) - w_s(1+\boldsymbol{g})^{s-t}\right](1-\boldsymbol{d})^{s-t-1} \Big/ \left(\prod_{t=t}^{s-1}(1+r_t)\right)\\
&+ p_{t+\overline{T}(t)+1}\Big\{\Delta T(t)\Big[(1-tr_{t+\overline{T}(t)+1})A_t F_2'(\boldsymbol{k}_t,1) - \left(w_{t+\overline{T}(t)+1}/p_{t+\overline{T}(t)+1}\right)(1+\boldsymbol{g})^{\overline{T}(t)+\Delta T(t)}\Big]\\
&- x^f_{t+\overline{T}(t)+1}(1+\boldsymbol{g})^{\overline{T}(t)+\Delta T(t)}\left(\frac{p_{T+\overline{T}(t)+1}}{p_{t+\overline{T}(t)}}\frac{1-\boldsymbol{d}}{1+r_{t+\overline{T}(t)}}\right)^{\Delta T(t)-1}\Big\}(1-\boldsymbol{d})^{\overline{T}(t)} \Big/ \left(\prod_{t=t}^{t+\overline{T}(t)}(1+r_t)\right) = 0
\end{aligned}\tag{6}$$

If capital intensity is eliminated between equations (5) and (6), we get a factor cost frontier which involves present and future interest rates and future wages rates.

Boucekkine et al. (2000) investigate the best way to solve numerically a vintage capital model. They start with a model written in continuous time and advise to approximate it in discrete time. We prefer to start with a model written in discrete time and to make intra-period approximations. They solve the discrete optimisation problem directly by an iterative method. We prefer to write the first order conditions, and to simulate the equations of the model including these conditions.

IV SCRAPPING AND AGGREGATION

We now consider the decisions concerning the production units built before period t. For each vintage of capital, its capital intensity being already set, the investor checks whether it is still profitable. If not, it is discarded.

Under our assumptions, a production unit is built during a period (which is an integer number), but can be used for the whole or part of a period. Let us call $\overline{a}(t)$ the age of the oldest unit which is used at the beginning of period t[11], i.e. at time $t-1$, and which will be scrapped at time $t-1+\Delta a(t) < t$. This unit was built at period $t-\overline{a}(t)$. If at this time future was perfectly foreseen, its value at the end of this period is:

$$\begin{aligned}
&V_{t-\overline{a}(t),t-\overline{a}(t)}(t,\Delta a(t)) =\\
&\sum_{s=t-\overline{a}(t)+1}^{t-1}\left[(1-tr_s)\,p_s A_{t-\overline{a}(t)}(1+g)^{t-\overline{a}(t)}F(k_{t-\overline{a}(t)},1) - w_s(1+g)^s\right](1-d)^{s-t+\overline{a}(t)-1} \Big/ \left(\prod_{t=t-\overline{a}(t)}^{s-1}(1+r_t)\right)_{\mathrm{T}}\\
&+ p_t\Big\{\Delta a(t)\Big((1-tr_t)A_{t-\overline{a}(t)}(1+g)^{t-\overline{a}(t)}F(k_{t-\overline{a}(t)},1) - (w_t/p_t)(1+g)^{t+\Delta a(t)-1}\Big)\\
&- x_t^f(1+g)^{t+\Delta a(t)-1}\left(\frac{p_t}{p_{t-1}}\frac{1-d}{1+r_{t-1}}\right)^{\Delta a(t)-1}\Big\}(1-d)^{\overline{a}(t)-1} \Big/ \left(\prod_{t=t-\overline{a}(t)}^{t-1}(1+r_t)\right)
\end{aligned}$$

o compute the date of scrapping we consider $t-\overline{a}(t)$ as given, and we assume the future to be perfectly foreseen. $t-1$ and $\Delta a(t)$ must be the integer part and the decimal part of the date of the scrapping. The conditions which must be satisfied by these two variables are:

[11] So, the unit has been active at this date for $\overline{a}(t)-1$ periods.

(7) $$\begin{cases} V_{t-\bar{a}(t),t-\bar{a}(t)}(t,\Delta a(t)) - V_{t-\bar{a}(t),t-\bar{a}(t)}(t+1,\Delta a(t)) > 0 \\ V_{t-\bar{a}(t),t-\bar{a}(t)}(t,\Delta a(t)) - V_{t-\bar{a}(t),t-\bar{a}(t)}(t-1,\Delta a(t)) > 0 \end{cases}$$

and:

$$\frac{\partial V_{t-\bar{a}(t)}(t,\Delta a(t))}{\partial \Delta a(t)} = 0$$

or:

(8) $$\left((1-tr_t)(1+g)^{-\bar{a}(t)-\Delta a(t)+1} A_{t-\bar{a}(t)} F(k_{t-\bar{a}(t)},1) - \left(\frac{w_t}{p_t}\right)\right) - \Delta a(t)\left(\frac{w_t}{p_t}\right)\ln(1+g)$$
$$- x_t^f \left(\frac{p_t}{p_{t-1}}\frac{1-d}{1+r_{t-1}}\right)^{\Delta a(t)-1}\left[\ln(1+g) + \ln\left(\frac{p_t}{p_{t-1}}\frac{1-d}{1+r_{t-1}}\right)\right] = 0$$

We can show:

Proposition 2 : In the neighbourhood of the steady state and for small growth rates, interest rates and depreciation rates, equation (8) determines a unique integer value for $\bar{a}(t)$ and a unique real value included between 0 and 1 for $\Delta a(t)$. These values satisfy inequations (7)
Proof : See Appendix 2.

It is now possible to define the aggregate level of employment and production. At date t, the production structure available is characterised by the series: $\left\{n_{t-a}(1-d)^a, k_{t-a}\right\}$, where a is the age of the different production units in working order ($1 \le a \le \bar{a}(t)$), and by $\Delta a(t)$. Aggregate employment and production capacity are obtained by summing over these vintages[12]:

(9) $$N_t = \sum_{a=1}^{\bar{a}(t)-1} n_{t-a}(1-d)^a + \Delta a(t) n_{t-\bar{a}(t)}(1-d)^{\bar{a}(t)}$$

12 We could add the assumption that once a productive unit was scrapped, it cannot be put in use again. In this case, we should introduce the constraint $\bar{a}(t) \le \bar{a}(t-1)+1$ in our optimisation problem. The maximum number of available units of production can be called the physical productive capacity. In general this capacity will not be saturated, the exception being a strong unanticipated increase in demand at time t.

The creation of employment at period t is n_{t-1} and the destruction of employment is: $-N_t + N_{t-1} + n_{t-1}$.

$$
\begin{aligned}
(10)\quad Y_t = & \sum_{a=1}^{\bar{a}(t)-1} A_{t-a}(1+g)^{t-a} F(k_{t-a},1) n_{t-a} (1-d)^a \\
& + \Delta a(t) A_{t-\bar{a}(t)} (1+g)^{t-\bar{a}(t)} F(k_{t-\bar{a}(t)},1) n_{t-\bar{a}(t)} (1-d)^{\bar{a}(t)}
\end{aligned}
$$

V CONCLUSION

A vintage capital model has interesting feature, which were investigated in Cadiou, Dées and Laffargue 2001), and before by Boucekkine (1997) and Boucekkine and al. (1998). First, such a model gives a replacement effect echo, that is cyclical answers to shocks, with a period equal to the expected lifetime of capital. This can be seen in Marmotte by its complex eigenvalues smaller than 1, with periods equal to this expected lifetime and to its harmonics. We have a similar property for the smallest eigenvalues larger than 1. Both sets of eigenvalues represent the Fourier decomposition of the two cycles related respectively to the backward side of the model (scrapping) and to its forward side (the building of new units).

Another feature of a vintage capital model is its ability to represent medium term changes in the distribution of income distribution, such as those empirically investigated by Blanchard (1997). Cadiou, Dées and Laffargue 2001) show, how an increase in the bargaining power of trade unions will increase the share of labour in national income, and how this increase will progressively disappear over time. They are also able to reproduce the dynamics of this share observed on the last 30 years in France, as previously Caballero and Hammour(1998) did. Syuch kinds of scenarios are easy to simulate with Marmotte.

Research on putty clay technology was popular in the 70's (e.g. Adachi (1974), Britto (1970), Calvo (1976)) and was at the center of the analysis of the consequences of the oil shock on factor demands in the beginning of the 80's. However, it was eventually more or less abandoned for its lack of tractability, especially under the hypothesis of rational expectations. Since the mid 90s, from the works by Caballero and Hammour (1994) and Boucekkine, Germain and Licandro. (1997), this research field has known a renewed interest for its ability to explain some major economic developments observed in industrialised countries over the last three decades. First, as putty-clay technology involves some stickiness in the production process, it enables to investigate properly the slow adjustment of production factors to shocks. Second, this framework also explicitly takes into account movements in job creation and job destruction related to economic obsolescence, replacement of productive capacity and expectations over the lifetime of the units.

The originality of the model proposed here is that it is written in discrete time whereas previous, recent works developed models in continuous time. Even under strong assumptions, it is almost impossible to derive the analytical solutions of continuous time models. The trick then usually consists in deriving discrete time formula from these models. Here, we prefer to start directly with a discrete time framework and use a second order relaxation algorithm to simulate the model. The traditional drawback of such a process was the presence of variables with long leads and long lags. However, progress in computation techniques has overcome these difficulties. For instance, by using the Stack algorithm implemented in Troll, the model can be easily solved.

APPENDIX 1

PROOF OF PROPOSITION 1

The expected life of a production unit built at time t is computed by maximising $V_{t,t}$ which will be denoted hereafter V, relatively to $\overline{T}(t)$, which will be denoted $\overline{T}$, and $\Delta T(t)$ which will be denoted ΔT.

For sensible values of horizon $\overline{T}$ we can assume that the expected state of the economy has reached a steady state. More precisely we will assume that for dates near this horizon, taxation rate tr, the real cost of labour in efficiency units $w/p = w$, real firing costs x^f and the expected real interest rate divided by 1 minus the depreciation rate $1+R$, are constant[13]. Thus, the function to maximise is (after the removal of an additive and a positive multiplicative constants):

$$V(\overline{T},\Delta T) = [(1-tr)AF - w(1+g)^{\overline{T}-1}] + [(1-tr)AF - w(1+g)^{\overline{T}}]/(1+R)$$
$$+\left\{\Delta T[(1-tr)AF - w(1+g)^{\overline{T}+\Delta T}] - x^f(1+g)^{\overline{T}+\Delta T}(1+R)^{1-\Delta T}\right\}/(1+R)^2$$

$\overline{T}$ must be a natural integer number and ΔT must be included between 0 and 1.

Maximisation relatively to ΔT for $\overline{T}$ given

We have to maximise:

$$\left\{\Delta T[(1-tr)AF - w(1+g)^{\overline{T}+\Delta T}] - x^f(1+g)^{\overline{T}+\Delta T}(1+R)^{1-\Delta T}\right\}$$

Let us cancel the derivative of this expression relatively to ΔT :

$$[(1-tr)AF(1+g)^{-\overline{T}-\Delta T} - w] - \log(1+g)w\Delta T - x^f(1+R)^{1-\Delta T}[\ln(1+g) - \ln(1+R)] = 0$$

We get the same equation as condition (4).

Let us assume : $R > g > 0$, with R, g small. Then, the first-order condition can be rewritten:

(A1.1) $$(1-tr)AF(1+g)^{-\overline{T}}(1+g)^{-\Delta T} - w(1+g)^{\Delta T} + x^f(R-g) = 0$$

For $\overline{T}$ given, the left-hand side of this equation is a decreasing function of ΔT. Let us look for a solution with ΔT included between 0 and 1. For such a solution to exist it is necessary and sufficient that the left-hand side of this equation is positive for: $\Delta T = 0$, and negative for: $\Delta T = 1$. Thus :

(A1.2) $$(1-tr)AF(1+g)^{-\overline{T}} - w + x^f(R-g) > 0 >$$
$$(1-tr)F(1+g)^{-\overline{T}}(1+g)^{-1} - w(1+g) + x^f(R-g)$$

[13] These assumptions can be slightly generalised as is indicated in Appendix 2.

Let us define the auxiliary variable t by equation :

$$(1-tr)AF(1+g)^{-t} = w - x^f (R-g)$$

Under the natural condition : $(1-tr)AF > w$, this equation defines a unique positive value t. Let us put : $\overline{T} = \text{integer}\,(t)$. Then, the left-hand side of inequation (A1.2) is satisfied. We also have : $0 > (1-tr)AF(1+g)^{-\overline{T}}(1+g)^{-1} - w + x^f (R-g)$. Then *a fortiori* the right-hand side of inequation (A1.2) is also satisfied.

Maximisation relatively to $\overline{T}$ for ΔT given

We must have: $V(\overline{T},\Delta T) > V(\overline{T}-1,\Delta T)$ and $V(\overline{T},\Delta T) > V(\overline{T}+1,\Delta T)$

or :

$$\begin{aligned}
&[(1-tr)AF - w(1+g)^{\overline{T}}]/(1+R) \\
\text{(A1.3)}\quad &+ \left\{\Delta T[(1-tr)AF - w(1+g)^{\overline{T}+\Delta T}] - x^f (1+g)^{\overline{T}+\Delta T}(1+R)^{1-\Delta T}\right\}/(1+R)^2 \\
&- \left\{\Delta T[(1-tr)AF - w(1+g)^{\overline{T}+\Delta T-1}] - x^f (1+g)^{\overline{T}+\Delta T-1}(1+R)^{1-\Delta T}\right\}/(1+R) > 0
\end{aligned}$$

and :

$$\begin{aligned}
&[(1-tr)AF - w(1+g)^{\overline{T}+1}]/(1+R)^2 \\
\text{(A1.4)}\quad &+ \left\{\Delta T[(1-tr)AF - w(1+g)^{\overline{T}+\Delta T+1}] - x^f (1+g)^{\overline{T}+\Delta T+1}(1+R)^{1-\Delta T}\right\}/(1+R)^3 \\
&- \left\{\Delta T[(1-tr)AF - w(1+g)^{\overline{T}+\Delta T}] - x^f (1+g)^{\overline{T}+\Delta T}(1+R)^{1-\Delta T}\right\}/(1+R)^2 < 0
\end{aligned}$$

We want to show that these two inequalities are satisfied if equation (A1.1) is satisfied and if: $0 \le \Delta T \le 1$. Condition (A1.3) can be rewritten:

(A1.5) $(1-tr)AF(1+g)^{-\overline{T}} - w + x^f (R-g) - \Delta T[(1-tr)AF(1+g)^{-\overline{T}} R - w(R-g)] > 0$

Condition (A1.4) can be rewritten:

(A1.6) $(1-tr)AF(1+g)^{-\overline{T}-1} - w + x^f (R-g) - \Delta T[(1-tr)AF(1+g)^{-\overline{T}-1} R - w(R-g)] < 0$

Equation (A1.1) can be rewritten:

$$(1-tr)AF(1+g)^{-\overline{T}} = w(1+g)^{2\Delta T} - x^f (R-g)(1+g)^{\Delta T}$$

By substitution of this equation in inequation (A1.5) we get:

$$[w(1+g)^{2\Delta T} - x^f (R-g)(1+g)^{\Delta T} - w](1-R\Delta T) + x^f (R-g) - wg\Delta T > 0$$

As R and g are small, this inequality can be rewritten:

$$[wg2\Delta T - x^f (R-g)] + x^f (R-g) - wg\Delta T > 0$$

or:

$wg\Delta T > 0$, which is satisfied because: $\Delta T > 0$.

By substitution of (A1.1) in inequation (A1.6) we get:

$$[w(1+g)^{2\Delta T-1} - x^f (R-g)(1+g)^{\Delta T-1} - w](1 - R\Delta T) + x^f (R-g) - wg\Delta T < 0$$

or:

$$[wg\,2\Delta T - wg - x^f (R-g)] + x^f (R-g) - wg\Delta T < 0$$

or:

$wg(\Delta T - 1) < 0$, which is satisfied because: $\Delta T < 1$.

Conclusion

The demonstration suggests the following simulation process of the model:

Fix $\overline{T}$ for each date of the simulation period (for instance at its reference steady state value) and introduce equation (4), but not inequations (3) in the simulation program.

If ΔT is not included between 0 and 1 for some periods, then compute: $\overline{T} = \text{integer } (\overline{T} + \Delta T)$ for these periods. Come back to step 1 and operate iteratively until convergence of the process.

If, for a period of the simulation, we get for a given value of the integer part equal to $\overline{T}$: $\Delta T < 0$, and for an integer part equal to $\overline{T} - 1$: $\Delta T > 1$, remove equation (4) for this period and keep $\overline{T}$ and $\Delta T = 0$.

Finally check for inequations (3).

We have no proof that this simulation process works. But, when we used it, we met no difficulty, and we conjecture that it works in the neighbourhood of the reference steady state.

APPENDIX 2
PROOF OF PROPOSITION 2

We will remove all reference to date t and to capital intensity when this will not introduce any ambiguity. We will put: $c_i = 1$, $w/p = \boldsymbol{w}$ and $1/(1+R_t) = \dfrac{p_{t+1}}{p_t}\left(\dfrac{1-\boldsymbol{d}}{1+r_t}\right)$. The function to maximise is (after the removal of an additive and a positive multiplicative constants):

$$V(\Delta a,t) = \sum_{s=t-\bar{a}+1}^{t-1}\left[(1-tr_s)AF - \boldsymbol{w}_s(1+\boldsymbol{g})^{s-t_0+\bar{a}}\right] \Big/ \left(\prod_{t=t-\bar{a}}^{s-1}(1+R_t)\right)$$
$$+\left\{\Delta a\left((1-tr_t)AF - \boldsymbol{w}_t(1+\boldsymbol{g})^{\bar{a}+\Delta a-1}\right) - x_t^f(1+\boldsymbol{g})^{\bar{a}+\Delta a-1}(1+R_{t-1})^{1-\Delta a}\right\} \Big/ \left(\prod_{t=t-\bar{a}}^{t-1}(1+R_t)\right)$$

Maximisation relatively to Δa

We have to maximise :

$$\Delta a\left((1-tr_t)AF - w_t(1+g)^{\bar{a}+\Delta a-1}\right) - x_t^f(1+g)^{\bar{a}+\Delta a-1}(1+R_{t-1})^{1-\Delta a}$$

Let us cancel the derivative of this expression relatively to Δa :

$$\left((1-tr_t)AF(1+g)^{-\bar{a}-\Delta a+1} - w_t\right) - \Delta a w_t \ln(1+g) - x_t^f(1+R)^{1-\Delta a}[\ln(1+g) - \ln(1+R_{t-1})] = 0$$

We get the same equation as condition (8).

Let us assume : $R > g > 0$, with R, g small. Then, the first-order condition can be rewritten:

(A2.1) $$(1-tr_t)AF(1+\boldsymbol{g})^{-\bar{a}-\Delta a+1} - \boldsymbol{w}_t(1+\boldsymbol{g})^{\Delta a} + (R_{t-1}-\boldsymbol{g})x_t^f = 0$$

For $\bar{a}$ given, the left-hand side of this equation is a decreasing function of Δa. Let us look for a solution with Δa included between 0 and 1. For such a solution to exist it is necessary and sufficient that the left-hand side of this equation is positive for: $\Delta a = 0$, and negative for: $\Delta a = 1$. Thus :

(A2.2) $$\begin{aligned}&(1-tr_t)AF(1+\boldsymbol{g})^{-\bar{a}+1} - \boldsymbol{w}_t + (R_{t-1}-\boldsymbol{g})x_t^f > 0 > \\ &(1-tr_t)AF(1+\boldsymbol{g})^{-\bar{a}} - \boldsymbol{w}_t(1+\boldsymbol{g}) + (R_{t-1}-\boldsymbol{g})x_t^f\end{aligned}$$

Let us define the auxiliary variable $\boldsymbol{a}$ by equation :

$$(1-tr_t)AF(1+g)^{-a+1} - w_t + (R_{t-1}-g)x_t^f = 0$$

Under the natural condition : $(1-tr_t)AF - \boldsymbol{w}_t > 0$, this equation defines a unique positive value $\boldsymbol{a}$. Let us put : $\bar{a} = \text{integer}\ (\boldsymbol{a})$. Then, the left-hand side of inequation (A2.2) is satisfied. We also have : $0 > (1-tr)AF(1+g)^{-\bar{a}}(1+g)^{-1} - w + (R_{t-1}-g)x^f$. Then *a fortiori* the right-hand side of inequation (A2.2) is also satisfied.

In Appendix 2, as F did not change with $\overline{T}$, equation $\overline{T} = \text{integer}\,(t)$ always had a solution. Now, capital intensity and F change with $\overline{a}$, and equation: $\overline{a} = \text{integer}\,(a)$, may have no solution or an infinity of solutions. However, by assuming to be in the neighbourhood of the steady state, we assume that capital intensity does not change too much across vintages, and we remove these possibilities.

Computation of $\overline{a}$

Conditions (7) are:

$$
\begin{aligned}
V = & \sum_{s=t-\overline{a}+1}^{-1}\left[(1-tr_s)AF - w_s(1+g)^{s-t+\overline{a}}\right] / \left(\prod_{t=t-\overline{a}}^{s-1}(1+R_t)\right) \\
& + \left\{\Delta a\left((1-tr_t)AF - w_t(1+g)^{\overline{a}+\Delta a-1}\right) - x_t^f(1+g)^{\overline{a}+\Delta a-1}(1+R_{t-1})^{1-\Delta a}\right\} / \left(\prod_{t=t-\overline{a}}^{t-1}(1+R_t)\right) > \\
& \sum_{s=t-\overline{a}+1}^{t-2}\left[(1-tr_s)AF - w_s(1+g)^{s-t+\overline{a}}\right] / \left(\prod_{t=t-\overline{a}}^{s-1}(1+R_t)\right) \\
& + \left\{\Delta a\left((1-tr_{t-1})AF - w_{t-1}(1+g)^{\overline{a}+\Delta a-2}\right) - x_{t-1}^f(1+g)^{\overline{a}+\Delta a-2}(1+R_{t-2})^{1-\Delta a}\right\} / \left(\prod_{t=t-\overline{a}}^{t-2}(1+R_t)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \sum_{s=t-\overline{a}+1}^{t-1}\left[(1-tr_s)AF - w_s(1+g)^{s-t+\overline{a}}\right] / \left(\prod_{t=t-\overline{a}}^{s-1}(1+R_t)\right) \\
& + \left\{\Delta a\left((1-tr_t)AF - w_t(1+g)^{\overline{a}+\Delta a-1}\right) - x_t^f(1+g)^{\overline{a}+\Delta a-1}(1+R_{t-1})^{1-\Delta a}\right\} / \left(\prod_{t=t-\overline{a}}^{t-1}(1+R_t)\right) > \\
& \sum_{s=t-\overline{a}+1}^{t}\left[(1-tr_s)AF - w_s(1+g)^{s-t+\overline{a}}\right] / \left(\prod_{t=t-\overline{a}}^{s-1}(1+R_t)\right) \\
& + \left\{\Delta a\left((1-tr_{t+1})AF - w_{t+1}(1+g)^{\overline{a}+\Delta a}\right) - x_{t+1}^f(1+g)^{\overline{a}+\Delta a}(1+R_t)^{1-\Delta a}\right\} / \left(\prod_{t=t-\overline{a}}^{t}(1+R_t)\right)
\end{aligned}
$$

The two inequations can be rewritten:

$$
\begin{aligned}
& \left[(1-tr_{t-1})AF(1+g)^{-\overline{a}+1} - w_{t-1}\right] \\
\text{(A2.3)} & + (1/(1+R_{t-1}))\left\{\Delta a\left((1-tr_t)AF(1+g)^{-\overline{a}+1} - w_t(1+g)^{\Delta a}\right) - x_t^f(1+g)^{\Delta a}(1+R_{t-1})^{1-\Delta a}\right\} > \\
& \left\{\Delta a\left((1-tr_{t-1})AF(1+g)^{-\overline{a}+1} - w_{t-1}(1+g)^{\Delta a-1}\right) - x_{t-1}^f(1+g)^{\Delta a-1}(1+R_{t-2})^{1-\Delta a}\right\}
\end{aligned}
$$

(A2.4)

$$
\begin{aligned}
& \left\{\Delta a\left((1-tr_t)AF(1+g)^{-\overline{a}+1} - w_t(1+g)^{\Delta a}\right) - x_t^f(1+g)^{\Delta a}(1+R_{t-1})^{1-\Delta a}\right\} > \\
& \left[(1-tr_t)AF(1+g)^{-\overline{a}+1} - w_t(1+g)\right] \\
& + (1/(1+R_t))\left\{\Delta a\left((1-tr_{t+1})AF(1+g)^{-\overline{a}+1} - w_{t+1}(1+g)^{1+\Delta a}\right) - x_{t+1}^f(1+g)^{1+\Delta a}(1+R_t)^{1-\Delta a}\right\}
\end{aligned}
$$

We can simplify inequation (A2.3) by putting: $\Delta w = (w_t - w_{t-1}) / w_t$: $\Delta x^f = (x^f{}_t - x^f{}_{t-1}) / x^f{}_t$ $\Delta tr = (tr_t - tr_{t-1}) / (1 - tr_t)$, and by assuming these three variations to be small :

(A2.5)

$$(1-tr_t)AF(1+g)^{-\overline{a}+1}(1+\Delta tr) - w_t(1-\Delta w) + x_t^f[R_{t-1} - g - (R_{t-1} - R_{t-2})(1-\Delta a) - \Delta x^f]$$
$$-\Delta a\left\{(1-tr_t)AF(1+g)^{-\overline{a}+1}[\Delta tr + R_{t-1}] - w_t[R_{t-1} - g - \Delta w]\right\} > 0$$

Equation (A2.1) can be rewritten :

$$(1-tr_t)AF(1+g)^{-\overline{a}+1} = w_t(1+2g\Delta a) - (R_{t-1} - g)x_t^f :$$

Let us substitute in inequation (A2.3):

$$w_t(\Delta w + \Delta tr + 2g\Delta a) - x_t^f[(R_{t-1} - R_{t-2})(1-\Delta a) - \Delta x^f] - \Delta a\left\{w_t[\Delta tr + g + \Delta w]\right\} > 0$$

or:

(A2.5) $$w_t[(\Delta w + \Delta tr)(1-\Delta a) + g\Delta a] - x_t^f[(R_{t-1} - R_{t-2})(1-\Delta a) - \Delta x^f] > 0$$

We can always find Δtr , $\Delta w, \Delta x^f$ et $-R_{t-1} + R_{t-2}$ small enough for inequation (A2.5) to be satisfied if: $w_t g\Delta a > 0$, *i.e.* if: $\Delta a > 0$

Inequation (A2.4) can be rewritten:

$$\left[(1-tr_t)AF(1+g)^{-\overline{a}+1} - w_t(1+g)\right] + x_t^f[R_t - g - \Delta x_{t+1}^f \Delta a)]$$
$$+\Delta a\left\{-(1-tr_{t_0})F(1+g)^{-\overline{a}+1}[\Delta tr_{+1} + R_t] + w_t[R_t - \Delta w_{+1}\Delta a - g]\right\} < 0$$

or, with equation (A2.1):

$$w_t g(\Delta a - 1) + x_t^f[R_t - R_{t-1} - \Delta x_{t+1}^f \Delta a)] + \Delta a w_t[-\Delta w_{+1}\Delta a - \Delta tr_{+1}] < 0$$

We can always find Δtr , $\Delta w, \Delta x^f$ et $-R_{t_0-1} + R_{t_0-2}$ small enough for inequation (A2.5) to be satisfied if: $w_t g(\Delta a - 1) < 0$, *i.e.* if: $\Delta a < 1$

The simulation process suggested at the end of Appendix 1 can easily be extended to the problem of this appendix.

CHAPTER II

CONSUMPTION BEHAVIOUR[14]

This chapter presents the consumption function of Marmotte and its estimation for the 17 countries present in the model. Macro-econometric studies on consumption often lie on extensions of the permanent income model. Even if we owe the concept of Permanent Income to Friedman (1957), the model used in the recent literature is due to Hall (1978). It implies that aggregated time series is interpreted as the solution to the infinite-lived representative consumer program. It considers a representative consumer who maximises the discounted sum of his instantaneous utilities. Taking his preferences into account, each individual chooses between consuming today and saving to consume later by comparing the effects of each of these two choices on his welfare.

The strong conclusion of the infinite-horizon model that changes in consumption follow a martingale difference was challenged by empirical studies. Hall's model seems to underestimate both the inertia of consumption relative to the permanent income (the 'excess smoothness' of consumption has been shown first by Deaton, 1988) and the sensitivity of consumption to the current income (Flavin, 1981, Campbell and Mankiw, 1989). This second limit is related to the existence of liquidity constraints that undermine the model's assumption of competitive financial markets. In spite of the financial deregulation implemented at some time in the past two decades in almost all the industrialised countries, part of the households may still be unable to borrow against their future income. Finally, in a theoretical point of view, the infinite life framework has also often been criticised for the lack of realism of its assumption concerning the agent's horizon, which avoids taking into account life cycle features and the distribution of income across generations.

An extension to a finite life horizon is due to Blanchard (1985). It enables to analyse some intergenerational distribution issues, such as the burden of public debt. The main feature of this approach is to account for the uncertainty that an individual agent faces relative to its life horizon. Although life expectancy is perfectly known, this uncertainty leads to unexpected bequests. With a perfectly competitive life insurance system, this introduces a distinction between the individual and the national rate of return. The real interest rate of the economy can then differ from each agent's time preference rate, within a range limited by the probability of dying. An important consequence of this framework is to rule out pure Ricardian equivalence: households anticipate that part of the burden of an increase in public debt will fall on younger households and future generations. However, the flexibility given by Blanchard's specification should not be overstated. Uncertainty about life horizon increases the private discount factor by the probability of dying. This gives some flexibility in setting the interest rate, which has not to be strictly equal to the time preference rate. But the rate of death is very low in industrialised countries (less than 0.5% per annum), so this flexibility and the departure from pure Ricardian equivalence are quantitatively limited. The other sources of non Ricardian equivalence seems to us more relevant, especially the imperfection on financial markets.

The implementation of Blanchard's style consumption function in a macro-econometric model is also not straightforward. First, the estimation of the model's parameters requires building data for unobserved variables, such as human wealth (i.e. the permanent income) and the

[14] This chapter is an adaptation of Allais, Cadiou and Dées (2001).

expectations of the future path of real interest rates. There is no trivial way to deal with this problem. As we know the motion law of the two unobserved data, it is tempting to compute the series by assuming starting values far enough in the past. However, one can be very sceptical about the use of "home-made" data in the estimation of the "deep" parameters of the economy. A natural way to avoid computing human capital is to estimate directly the arbitrage equation of the consumer. But it is not possible to do so in Blanchard's framework because this equation does not hold at the aggregate level, although it does for individual agents.

Considering that the empirical costs exceed the economic benefits of the finite life model, we have decided to choose for Marmotte a more traditional framework based on an extension of the Hall's model. Extending the infinite-horizon model is related with the use of the capital asset pricing model, which considers that the individual has access to complete financial markets without transaction cost. Hence, any type of financial asset can be used as a means of saving. The arbitrage condition builds up a relationship between the asset's expected return and the marginal rate of intertemporal substitution, i.e. the relative importance given by the individual between consuming today and consuming at the next period. Assuming an infinite-horizon framework allows us to preserve its convenience and its tractability in terms of econometric estimation. Furthermore, we attempt to deal with the two empirical limits of the Hall's model: (a) excess smoothness of consumption relative to permanent income and (b) liquidity constraint.

To account for the excess smoothness of consumption, we reconsider the assumption of a time separable utility function to take into account habit formation. Following Weil (1989) and Constantinides (1990), we expect to enhance the ability of the model to explain consumption inertia. We derive an arbitrage condition from an iso-elastic instantaneous utility function with current and past consumption as arguments.

This liquidity constraint effect is difficult to integrate in a theoretical model in a tractable way. More precisely, the heterogeneity across agents regarding their financial wealth makes it impossible to derive any micro-based macro-economic relation. A practical solution consists in assuming two different types of households whose proportion in the economy is constant over time. The households of the first group are liquidity-constrained. Although they want to borrow, they find no counterpart on the financial market. This means that they consume all their current income. The households of the second group have a free access to financial markets and behave according to the arbitrage equation. By including the liquidity constraints directly in the arbitrage equations, we want to derive the share of constrained households from the econometric estimation.

The theoretical framework of the behavioural equations retained for Marmotte is presented in section I. Then, section II displays the estimation of the parameters and discusses the relevance of country-specific values for the 17 countries modelled in Marmotte. Section III concludes.

I THEORETICAL FRAMEWORK

I.1 Habit formation in the consumers' behaviour

We consider here the approach of a representative agent with an infinite life horizon. The theoretical base of what follows is related to the consumption based capital asset pricing model (C-CAPM) theory as developed for instance by Weil (1989) and Constantinides (1990). These models have been motivated by the inability of the traditional model with a time separable utility function to explain observed risk premia (problem known as the " equity premium puzzle ", see Mehra and Prescott, 1985). The C-CAPM requires an unwisely high risk aversion coefficient to make up for the low volatility of consumption growth relative to the equity premium. This 'equity premium puzzle' has led some economists to question the specification of the model, in particular the time-separability of the representative agent's utility. Relaxing the hypothesis of time separable utility induces to extend the temporal effects of the consumption realised in a given period to the intertemporal utility of the consumer. We consider here the simple case where the present consumption has also an impact on the utility of the next period. The assumption of a time dependent utility function gives some flexibility to the model. More precisely, the impact of current consumption on future instantaneous utility reflects the formation of habits. Besides, this specification should enhance the ability of the model to explain consumption inertia (Fuhrer, 2000).

We assume an economy with a representative agent who chooses his consumption path so as to maximise the expected discounted sum of instantaneous utilities under his budget constraint. He has access to complete financial markets and holds *n* different assets. We define the instantaneous utility as a function of current and lagged (one period) consumption. This is a convenient way to break time separability. If lagged consumption depresses the current utility, consumption is characterised by habit formation: the representative agent gauges his utility partly by considering his previous level of consumption as a benchmark. On the contrary, if lagged consumption has a positive effect on the current utility, consumption is characterised by durability[15].

Two other parameters enter the consumption function: the time discount rate and the concavity of the instantaneous utility function. The interest of such a model is that the inter-temporal elasticity of substitution is no more equal to the inverse of risk aversion, but depends also on the habit (or durability) parameter.

The maximisation program is as follows:

$$(1)\ \mathrm{Max}\, E_t H_t = E_t\left[\sum_{t=0}^{+\infty} \boldsymbol{b}^t U\left(C_{t+t} - \boldsymbol{a}C_{t+t-1}\right)\right]$$

$$(2)\ pc_{t+t} C_t + \sum_{i=1}^{n} p_{i,t+t} S_{i,t+t+1} - \sum_{i=1}^{n}\left(p_{i,t+t} + d_{i,t+t}\right) S_{i,t+t} - YD_{t+t} = 0, \quad \forall \boldsymbol{t} \geq 0$$

where C_t is the level of the per capita consumption in real terms and pc_t the price of a unit of consumption at time t. E_t is the expectation of the representative agent conditional to the information set available at time t, $\boldsymbol{b}$ is the current discount factor, $\boldsymbol{a}$ is the habit parameter (if

[15] The habit formation model is presented in more details in Allais *et al.* (2000).

0<a<1) or the durability parameter (if –1<a<0). YD_t is the non-financial consumer's nominal disposable income. We can notice that habits depend on the past consumption realised *by the agent*, and not on the average level of past consumption in the economy as a whole[16]. When determining the level of its current consumption, the consumer takes into account not only the immediate satisfaction he gets from it, but also the impact these expenses have on its satisfaction in the next period.

The consumer holds a portfolio constituted by *n* assets. We note $S_{i,t}$ the number of assets i (i = 1, ..., n) bought in t-1 by the agent and held until time t, $p_{i,t}$ the price of the asset i in t and $d_{i,t}$ the amount of interest, coupon or dividends paid for each unit of the asset *i* hold between t-1 and t. Each asset has a return $R_{i,t}$. Among these *n* assets, the first one (i=1) is a risk-free asset whose return $R_{1,t}$ is certain. The *n-1* other assets are risky.

The first-order condition is:

(3) $$E_t\left[\frac{\partial H_t}{\partial C_{t+1}} / \frac{\partial H_t}{\partial C_t}\left[1+R_{i,t+1}\right]\right]=1 \quad \forall i = 2,..., n$$

where $$1+R_{i,t+1} = \left(\frac{pc_t}{pc_{t+1}}\right)\left(\frac{p_{i,t+1}+d_{i,t+1}}{p_{i,t}}\right)$$

with an iso-elastic utility function of the following form:

$$U(C_t - aC_{t-1}) = \frac{(C_t - aC_{t-1})^{1-g}}{1-g}$$

the first order condition (3) becomes:

(4) $$E_t\left[(C_t - aC_{t-1})^{-g} - b(a+(1+R_{i,t+1}))(C_{t+1}-aC_t)^{-g} + ab^2(1+R_{i,t+1})(C_{t+2}-aC_{t+1})^{-g}\right]=0$$
$$\forall i = 1,\dots,n$$

Equation (4) gives the form of the Euler equation, which reflects the consumption behaviour of not financially constrained households.

I.2 How to deal with the liquidity constraint issue?

Introducing liquidity constraints in the habit model can be realised quite easily. As Adda and Boucekkine (1996), we can re-write the maximisation program by modifying the constraint (2). Liquidity constraints can then be included just by adding a simple form imposing that the consumer cannot borrow if its financial wealth is above a threshold level $\overline{W}$:

(2') $$\begin{cases} \sum_{i=1}^{n} p_{i,t}S_{i,t+1} = \sum_{i=1}^{n}\left(p_{i,t}+d_{i,t}\right)S_{i,t} + YD_t - pc_t C_t \\ \sum_{i=1}^{n} p_{i,t}S_{i,t+1} > \overline{W} \qquad \forall i = 1,...,n \end{cases}$$

[16] This Class of model is referred to as « catching-up with the Joneses » (Abel, 1990).

The last inequality is a simple form for the liquidity constraint. However, due to the non-linearity and non-differentiability of the Euler equations, it is not possible to derive closed-form decision rules for optimal consumption. The model can only be solved using numerical simulations. It seems then difficult to identify the different characteristics of this general model, in particular between time non-separability and liquidity constraints (Adda and Boucekkine, 1996).

Yet, we need a specification for the consumption function that can be econometrically estimated. Consequently, we turn to an *ad hoc* specification, which aims at adding the liquidity constraint effect to the arbitrage equation with habit formation (equation 4). We suppose that a constant share of households faces a liquidity constraint. This simple assumption corresponds to a very specific form of the liquidity problem. With two types of consumers, we implicitly rule out the possible movements from one group to the other. This means that each individual consumer is either always or never liquidity constrained over his lifetime. This assumption could be criticised since agents may only face a liquidity constraint at the beginning of their life. Assuming constant flows from one group to the other has more appeal. However, under this alternative assumption the proportion of households facing a constraint would depend on the age structure of the population. There are also reasons to expect the liquidity constraint to be correlated to the business cycle, due for example to credit channel mechanisms.

Taking into account a time varying share of constrained agents would be both difficult and fragile. Here, as Campbell and Mankiw (1989, 1991), we consider a constant share of constrained households. However, these authors estimate a liquidity-constraint effect assuming a quadratic, time separable utility function, and thus taking advantage of the linearity of marginal utility of consumption. Indeed, in this case the change in the consumption of unconstrained households equals the expectation error on their permanent income, which is orthogonal to the information set of the agent. Thus, the share of consumption change explained by the change in current income can be assimilated to the share of constrained agents.

The transposition of this strategy to an iso-elastic utility function, also proposed by Campbell and Mankiw, has been considered but it has appeared too demanding. More precisely, the linearisation of Euler equations such as (3) rests upon the log-normality assumption of the conditional distribution of consumption and of financial asset returns.

Using the following transformations: $r_{i,t+1} = \ln(1 + R_{i,t+1})$ and $m_{t+1} = \ln\left(\frac{\partial H_t}{\partial C_{t+1}} / \frac{\partial H_t}{\partial C_t}\right)$,

equations (3) become:

$$(3') \qquad E_t\{r_{i,t+1}\} + E_t\{m_{t+1}\} + \frac{1}{2}\left[\sigma_t^2\{r_{i,t+1}\} + \sigma_t^2\{m_{t+1}\} + 2Cov_t\{r_{i,t+1}, m_{t+1}\}\right] = 0$$

where σ_t^2 and Cov_t are respectively the variance and the covariance conditional to the information set.

Assuming that $E_t\{m_{t+1}\}$ can be expressed as a linear combination of Δc_{t-1}, Δc_t, Δc_{t+1} and Δc_{t+2}, where $c_t = \ln(C_t)$, we would still have to ignore the variance and co-variance terms of equation (3') in order to estimate the parameters of the model. From our attempts to do so, it appears that this last assumption is much too strong. For example, without habit formation, i.e. with $\alpha = 0$ and $E_t\{m_{t+1}\} = \gamma E_t\{\Delta c_{t+1}\}$, the estimation of (3') gives very high values for γ, whereas the estimation of the genuine non-linear specification (3) gives more reasonable values for this parameter. We have the same result with habit formation, although in that case we also need to deal with the non-linearity of $E_t\{m_{t+1}\}$[17].

We propose here another way to estimate the share of liquidity constrained households in the economy. We assume that unconstrained households observe the behaviour of constrained households. The rationality of unconstrained households rests upon the fact that they know the working of the whole economy. In particular, they are aware that liquidity-constrained households consume their current income and that these households represent a share λ of the economy (in particular, they receive a share of the aggregated disposable income equal to λ).

Thus, the maximisation program of unconstrained households becomes:

(8) $$\text{Max}\, E_t H_t = E_t\left[\sum_{\tau=0}^{+\infty} \beta^\tau U\left(C^u_{t+\tau} - \alpha C^u_{t+\tau-1}\right)\right]$$

(9) with

$$\begin{cases} pc_{t+\tau} C_{t+\tau} + \sum_{i=1}^{n} p_{i,t+\tau} S_{i,t+\tau+1} - \sum_{i=1}^{n}\left(p_{i,t+\tau} + d_{i,t+\tau}\right) S_{i,t+\tau} - YD_{t+\tau} = 0 & \forall \tau \geq 0 \\ C_t = C_t^u + \lambda YD_t & \end{cases}$$

where C_t^u is the consumption of unconstrained households and λYD_t that of constrained households.

This gives the new Euler equations (10):

$$E_t\Big\{\big((C_t - \lambda YD_t) - \alpha(C_{t-1} - \lambda YD_{t-1})\big)^{-\gamma} - \beta(\alpha + (1+R_{i,t+1}))\big((C_{t+1} - \lambda YD_{t+1}) - \alpha(C_t - \lambda YD_t)\big)^{-\gamma}$$
$$+ \alpha\beta^2(1+R_{i,t+1})\big((C_{t+2} - \lambda YD_{t+2}) - \alpha(C_{t+1} - \lambda YD_{t+1})\big)^{-\gamma}\Big\} = 0$$
$$\forall i = 1,\ldots,n$$

I.3 Implementation in Marmotte

In Marmotte, the representative agent allocates its financial wealth according to four different types of assets: a one period, risk free asset, a long term asset related with government bonds, a domestic risky asset (the domestic firms' capital) and a foreign asset. The Euler equation (4) should be written for each asset, each equation linking the expected marginal substitution rate of consumption to the expected return of this asset. Thus, the estimation of the preference parameters should be based on the stochastic system consisting of four arbitrage equations. To simplify, in Marmotte, we have preferred to write only the Euler equation related to the risk-free asset's return. This simplification is theoretically justified in the case there is no uncertainty. Hence, without uncertainty, which is the case in deterministic macro-models, the system of the four Euler conditions can be reorganised. It is equivalent to a system consisting in one of the previous Euler equation for a specific asset and three relations setting the expected return of

[17] This has been done considering the Taylor development of $E_t\{m_{t+1}\}$ around a steady state.

this asset equal to the expected return of the other types of assets. In Marmotte, we have such arbitrage relations between the risk-free interest rate and the other asset's returns. The interest rate term structure relates the risk-free interest rate and the long-term government bond's return. The firms' capital returns are equal to the risk-free rate augmented by a risk premium defined by the factor cost frontier. And the interest rate related with foreign assets includes also an *ad hoc* risk premium that depends on the level of the foreign debt as a percentage of GDP.

For the estimation, we have used two arbitrage equations, the first one related with the risk-free asset and the second one related with the government bond. Data availability problems for stock returns for the 17 countries of Marmotte, and the low share of foreign assets in the households' financial wealth have led to the removal of the other two equations[18].

II ESTIMATING THE PARAMETERS OF THE CONSUMPTION FUNCTION

As we have previously seen, the specification of consumption behaviours in Marmotte allows the existence of two different types of consumers. The first one behaves according to an Euler equation. The second one faces a liquidity constraint and spends all its current income. In this section, we provide estimates for the parameters of equation (10) with the methodology developed in Allais, Cadiou and Dées (2000). These estimates will be used to parameterise the consumption function of the 17 countries modelled in Marmotte. Besides, they will be used to study the structural differences in consumption behaviours across those countries. With the model developed here, these differences are likely to come both from consumers' preferences (risk aversion, time preference, habit) and from market imperfection (liquidity constrained households).

II.1 The system of Euler equations

The consumption parameters are estimated from the two-equation system made of equations (10) written for the short-term asset (with return R_1) and bonds (with return R_2)[19]:

$$E_t\Big\{\big((C_t - \lambda YD_t) - \alpha(C_{t-1} - \lambda YD_{t-1})\big)^{-\gamma} - \beta(\alpha + (1+R_{i,t+1}))\big((C_{t+1} - \lambda YD_{t+1}) - \alpha(C_t - \lambda YD_t)\big)^{-\gamma}$$
$$+ \alpha\beta^2(1+R_{i,t+1})\big((C_{t+2} - \lambda YD_{t+2}) - \alpha(C_{t+1} - \lambda YD_{t+1})\big)^{-\gamma}\Big\} = 0$$

for i=1,2.

We have to estimate 4 parameters: the habit parameter (α), the discount factor (β), the concavity of the utility function (γ), and the share of liquidity constrained agents (λ). The estimations are realised assuming that unconstrained consumers have access to both bonds and money market asset.

II.2 The database

[18] An analysis of the habit consumption model with stock returns for the G7 countries is provided by Allais *et al.* (2000).

[19] The long-term bond is considered as a proxy of a perpetual bond whose return is defined as follows: $i_t^l + \frac{i_{t-1}^l}{i_t^l}$, where i_t^l is the long-term interest rate.

The estimations are realised with yearly series over the period starting in 1971 and ending in 1998. The database includes the 17 countries of Marmotte. The variables required for the estimations are *per capita* consumption (in nominal and real terms), disposable income, short-run interest rates and long-run interest rates. Households' consumption corresponds to the definition of the OECD Economic Outlook. The consumption deflator is obtained by dividing consumption in nominal terms by consumption in real terms. Data for disposable income are derived from households' saving ratio reported in the OECD Economic Outlook. Short-run interest rates are generally money market rates or 3-months Treasury bill rates as reported in the OECD Economic Outlook. As this series is partially missing for Spain in the early 70s, we have used the Bank of Spain intervention rate, which is very close to the OECD data during the period where the two series are available. For the long run interest rate, the OECD Economic Outlook series have been used for all the countries. These series usually refer to the 10-year government bonds.

II.3 Specific problem for the estimation of the consumption function

Identification

The use of the GMM requires first, the parameters to be identifiable and, second, the variables to be stationary (Hansen 1982).

The problem of identification was discussed in Allais, Cadiou and Dées (2000). We only summarise here the transformation we must make to overcome the identification issue. The main problem of the estimation of equation (10) is that $\gamma = 0$ is a trivial solution, since the objective function to be minimised is equal to zero in that case. Indeed, when $\gamma = 0$, the Euler equation simplifies to: $(1-\alpha\beta)E_t\left[1-\beta(1+R_{t+1})\right]=0$. Hence, any couple of values of α and β verifying $\alpha = 1/\beta$ is a solution. As a result, one of these parameters cannot be determined. This indeterminacy problem is not critical since the case $\gamma = 0$ has no economic sense in a model with habit formation. More precisely, we are interested in a solution within the class of strictly concave utility functions.

How to constraint γ to be strictly positive? Traditionally (see Allais, 1999 or Ogaki, 1993) the Euler equation is divided by $(1-\alpha\beta)\left[1-\beta(1+R_{t+1})\right]$. Then the objective function takes large values when the parameters approach to $\alpha\beta = 1$ and $\beta(1+E_t\{R_{t+1}\})=1$, rejecting these combinations as solutions. In our opinion, this method has serious drawbacks since this *ad hoc* modification has in practice a strong influence on the parameters that minimise the objective function.

The method chosen in this paper (explained in more details in Allais *et al*., 2000) avoids modifying too much the objective function. As β must not exceed 1 and $|\alpha| \leq 1$, there is only one evident solution which is $\alpha = 1$ and $\gamma = 0$. As we are interested in a model with habit formation, the case where $\gamma = 0$ has no relevance for us even if it corresponds to the global minimum of the objective function. In other words, we concentrate on the class of utility functions that are strictly concave, by searching the best local minimum that satisfies $\gamma > 0$. Practically, rather than estimating directly γ, we estimate a parameter θ such as $\gamma = \exp(\theta)$. Hence, we start to investigate solutions whose initial values are sufficiently far away from the evident solution. Finally, to ensure that λ remains positive, we also use the following variable change: $\lambda = \exp(\tau)$, and we estimate the parameter τ.

Stationarity

The second problem concerns the stationarity condition. Per capita consumption in the 17 countries does not satisfy the stationary condition, even though unit root tests are not powerful over such a small sample (28 years). To deal with this problem, we divide equation (10) by $\left((C_t - \lambda YD_t) - \alpha(C_{t-1} - \lambda YD_{t-1})\right)^{-\gamma}$, which involves estimating the following equation:

$$E_t\left\{1 - \beta\left(\alpha + (1 + R_{i,t+1})\right)\left(\frac{(C_{t+1} - \lambda YD_{t+1}) - \alpha(C_t - \lambda YD_t)}{(C_t - \lambda YD_t) - \alpha(C_{t-1} - \lambda YD_{t-1})}\right)^{-\gamma} + \alpha\beta^2\left((1 + R_{i,t+1})\right)\left(\frac{(C_{t+2} - \lambda YD_{t+2}) - \alpha(C_{t+1} - \lambda YD_{t+1})}{(C_t - \lambda YD_t) - \alpha(C_{t-1} - \lambda YD_{t-1})}\right)^{-\gamma}\right\} = 0 \tag{10'}$$

II.4 Estimation results

Estimation results of the different models and tests

Two different Euler equations are estimated simultaneously, each being related to a different asset return (short term and long term interest rates as defined by the OECD Economic Outlook). The theoretical model that we want to estimate has four parameters: γ the curvature of the instantaneous utility function, α the habit parameter, β the discount factor and λ the share of constrained households. This model is estimated for the 17 countries. We want to test the existence of significant differences in the value of the parameters across countries. The combinations of constraints on the parameters imply to estimate 16 different models. We present in Appendix 1 the results of these estimations. To guide our choice among the 16 models, we have implemented non-nested tests as explained in details in Laffargue and Guichard (2000). We test as the null hypothesis the equality of these parameters across countries by proceeding from the general to the particular. The procedure is the following. For each parameter, we estimate the model where this parameter is identical across countries. Then, we estimate the model where all the parameters are country-specific by using the covariance matrix of the previous model. We realise a likelihood ratio test (as defined by Ogaki, 1993) with, as the null hypothesis, the constrained model and, as the alternative hypothesis, the unconstrained model. For each test, we accept the null hypothesis at the 5 percent significance level.

If the constraint across the 17 countries is accepted for one parameter, then the model where this parameter is identical across countries becomes the reference model relative to which the equality of each of the two other parameters is tested. This procedure is continued until the equality constraint across countries is rejected for all the remaining unconstrained parameters. We have then the final model to retain.

If the estimation of all the models has been realised, most of the results exhibit unreasonable values in an economic viewpoint. For several models, the GMM algorithm tends to solutions where the curvature of the utility function is infinite. Besides, the parameter λ (the share of constrained agents), is sometimes larger than one. After having run the series of nested tests, we reached the conclusion that the best model retains identical parameters across the 17 countries. However, the tests between the most constrained model and the models for which only one parameter is country specific are unfair. Actually, the most constrained model is close to a trivial solution where $g = 0$ and $a = 1$, the other parameters being undetermined. It however stops before reaching this solution (at $g = 0.03$ and $a = 0.92$), owing to the non-negativity constraint of $(C_t - lYD_t) - a(C_{t-1} - lYD_{t-1})$. In other words, when all the parameters are the same across countries, there is no solution in the class of strictly concave utility function. However, in that case, the corresponding likelihood function used for the test is almost equal to zero making this uninteresting model the best one. Then, we must look at the models where one parameter is country-specific. In that case, the best model is the model where habit is country-specific. The estimates of this model are displayed in Table 1. This model performs quiet well since it would not be rejected at the 11% level against the “unfair” constrained model. Besides, the estimated values are reasonable in an economic viewpoint. This model is the one retained for Marmotte.

The estimations exhibit habit formation, i.e. positive values for α. This coefficient is significant for all the countries but Canada, Sweden and the UK. The discount factor is highly significant and the share of the constrained consumers is significant at the 10% level. It is equal to 13 %. This share has been estimated in several papers. One of the first estimations was realised by Campbell and Mankiw (1991). They found a share for the G7 countries ranging between 22% for the UK and 65% for Germany. These estimation were realised for periods starting in the 50s or 60s according to the countries and ending in 1986. These estimates are largely higher than ours. However, more recent studies all reckon a significant decrease in the share of constrained consumers since the financial liberalisation of the 80s. Patterson and Pesaran (1992) find 0.21 as an estimate of liquidity constrained consumption for the UK and 0.44 for the US over the period 1955-89. They also find that this share has fallen significantly in the 80s, to 0.13 for the UK and to 0.1 for the US. The latter estimates are in line with other empirical evidence of the decline in liquidity constrained consumption following the 80s financial liberalisation (e.g. Sefton and In't-Veld, 1999).

Table 1: Estimation results

	Value	*t-Stat*
Common parameters		
g (curvature of the utility function)	0,84	1,1
l (share of constrained agents)	0,13	1,6
b (discount factor)	0,96	75,0
***a* (Habit parameter)**		
Austria	-0,74	-8,7
Belgium	-0,64	-3,1
Canada	-0,20	-0,4
Denmark	-0,83	-2,7
Finland	-0,63	-1,7
France	-0,70	-4,8
Germany	-0,63	-3,4
Greece	-0,94	-21,9
Italy	-0,64	-4,2
Ireland	-0,82	-6,0
Japan	-0,97	-13,5
Netherlands	-0,63	-2,8
Portugal	-0,69	-6,1
Spain	-0,73	-6,6
Sweden	-0,39	-1,4
UK	-0,82	-1,5
US	-0,81	-5,1

Interpretation of the results for the unconstrained consumers

Here, we assess to what extent the estimated parameters reveal differences across countries in the preferences of the unconstrained households. If in a time-separable utility function, the parameters are directly interpretable in terms of relative risk aversion or (inverse of) elasticity of intertemporal substitution, this is not the case in our specification. Hence, we have explicitly computed the expressions of the elasticity of intertemporal substitution (EIS) and the coefficient of relative risk aversion (RRA).

The elasticity of intertemporal substitution summarises the consumer behaviour in the face of uncertainty on the level of consumption. It is defined by:

$$1/EIS_t = -\frac{C_t \dfrac{\partial^2 V_t}{\partial^2 C_t}}{\dfrac{\partial V_t}{\partial C_t}}$$, where V_t is the intertemporal utility function.

In our case, this is equivalent to:

$$1/EIS_t = \gamma \frac{C_t \left[(C_t - \alpha C_{t-1})^{-\gamma-1} + \beta\alpha^2 (C_{t+1} - \alpha C_t)^{-\gamma-1} \right]}{(C_t - \alpha C_{t-1})^{-\gamma} - \beta\alpha (C_{t+1} - \alpha C_t)^{-\gamma}}$$

Following Lettau and Uhlig (1997), we derive the expression of the elasticity of inter-temporal substitution by considering that the logarithm of consumption follows a random walk with drift: $c_{t+1} = g + c_t + \varepsilon_{t+1}$. This assumption simplifies the computation of the conditional expectations and gives an indication of the sensitivity of the elasticity of intertemporal substitution to the model's parameters:

$$1/EIS = \left(\frac{\gamma}{1 - \alpha e^{-g}} \right) \left(\frac{1 + \beta\alpha^2 e^{-g(\gamma+1)}}{1 - \beta\alpha e^{-g\gamma}} \right)$$

We find then that without habit formation (α=0), the inverse of the elasticity of intertemporal substitution is equal to the curvature of the instantaneous utility function. The elasticity of intertemporal substitution is a decreasing function of both habit and the curvature of the utility function. It also decreases with the discount factor β as soon as we have habit (α>0).

Relative risk aversion summarises the consumer's behaviour in the face of uncertainty on wealth:

$$RRA_t = -\frac{W_t \dfrac{\partial^2 V_t}{\partial^2 W_t}}{\dfrac{\partial V_t}{\partial W_t}}$$, where W_t is the wealth of the representative agent.

Constantinides (1990) gives the expression of relative risk aversion in the case of a production economy in which the agent's wealth is endogenous. We take here the formula in Lettau and Uhlig (1997), again in the case where the logarithm of consumption follows a random walk with drift:

$$RRA = \frac{\gamma}{1 - \alpha e^{-g} \dfrac{e^{-g\gamma} - \beta\gamma}{e^{-g\gamma} - \beta\alpha}}$$

The relative risk aversion decreases strongly with the degree of habit. On the other hand, the higher the habit coefficient is, the less relative risk aversion is sensitive to the curvature of the utility function (γ)

Because the consumption growth rate is not volatile enough, we have seen previously that the elasticity of intertemporal substitution should be very low to explain the level of the equity premium. Without habit, the relative risk aversion of the representative agent takes then values that are unreasonable. The advantage of the habit model is that it does not impose an equality constraint between relative risk aversion and the inverse of the elasticity of intertemporal substitution. In particular, Constantinides (1990) shows that, with habit formation, the product RRA x EIS is below one. This indicates that for the same elasticity of intertemporal substitution, relative risk aversion is weaker in a model with habit. The economic interpretation of this inequality is that the consumer smoothes its consumption more than is required by life cycle consideration. In fact:

$RRA_t \times ESI_t = \dfrac{\partial C_t}{\partial W_t}\dfrac{W_t}{C_t} < 1$ We find then an elasticity of consumption relative to wealth that is less than one.

With these formulas and the values of α, β and γ derived from our estimations, we can compute the values of RRA and EIS for our preferred model. We also assume that consumption (in logarithm) follows a random walk with a drift that is country-specific[20]. Even if this hypothesis on the consumption growth process is quite strong, it allows us to derive easily values required to compare the consumption behaviour across countries.

Table 2 below presents the values of the elasticity of intertemporal substitution (EIS) and of the relative risk aversion (RRA) according to the formula presented above.

The model gives interesting consumers' preferences. The low values of the elasticity of intertemporal substitution are consistent with the assumption that agents favour a very important smoothing of their consumption over time, although it takes extreme values for Greece and Japan. Without habit, these low elasticities of intertemporal substitution would lead to very high relative risk aversion. Here, coefficients for the relative risk aversion are reasonable. They range between 0.88 for Canada and 3.01 pour the UK[21].

The consumption models with habit formation are characterised by an excess smoothing of consumption relative to that implied by the life cycle hypothesis (Constantinides, 1990). The product $RRA \times EIS$, equal to one for the time separable models and less than one here, gives a measure of this excess smoothing. Our estimations indicate that the presence of habit implies a very low change of consumption relative to a change in wealth, in a ratio of 1 to 10 for most of the countries (Austria, Belgium, France, Italy, Portugal, Spain, the UK and the US). This relative change of consumption to wealth is a bit higher for Denmark, Finland, Germany, Ireland, and the Netherlands (1 to 6). The especially low excess smoothing for Sweden and Canada is a particular case and should be taken very cautiously, since the estimates for the habit coefficient are badly estimated in these cases.

[20] The drift is computed as being the average of $\log(C/C_{-1})$ over the estimation period.

[21] Note that the approximations underlying the formula for RRA do not apply for values of habit close to one, as shown in the case of Japan and Greece.

Table 2: Elasticity of Intertemporal Substitution and Relative Risk Aversion

	γ	α	β	Drift	**RRA**	**1/EIS**	**EIS x RRA**
Austria	0.84	0.74	0.96	0.024	**1.57**	**14.97**	**0.10**
Belgium	0.84	0.64	0.96	0.021	**1.20**	**7.83**	**0.15**
Canada	0.84	0.20	0.96	0.019	**0.88**	**1.34**	**0.66**
Denmark	0.84	0.83	0.96	0.017	**1.93**	**11.26**	**0.17**
Finland	0.84	0.63	0.96	0.020	**1.18**	**7.41**	**0.16**
France	0.84	0.70	0.96	0.021	**1.38**	**11.47**	**0.12**
Germany	0.84	0.63	0.96	0.018	**1.19**	**7.48**	**0.16**
Greece	0.84	0.94	0.96	0.023	**-0.81**	**162.94**	**0.00**
Ireland	0.84	0.64	0.96	0.028	**1.19**	**7.64**	**0.16**
Italy	0.84	0.82	0.96	0.025	**3.00**	**29.59**	**0.10**
Japan	0.84	0.97	0.96	0.027	**-0.34**	**311.91**	**0.00**
Netherlands	0.84	0.63	0.96	0.018	**1.18**	**7.46**	**0.16**
Portugal	0.84	0.69	0.96	0.022	**1.33**	**10.63**	**0.13**
Spain	0.84	0.73	0.96	0.021	**1.52**	**14.11**	**0.11**
Sweden	0.84	0.39	0.96	0.009	**0.95**	**2.49**	**0.38**
UK	0.84	0.82	0.96	0.024	**3.01**	**29.81**	**0.10**
US	0.84	0.81	0.96	0.019	**2.65**	**27.84**	**0.10**

II.5 Conclusion

This chapter has aimed at defining the consumption function of the multi-country model Marmotte and at estimating econometrically its parameters. We have assumed an infinite-horizon framework by extending the permanent income model. It has allowed us to preserve the tractability in terms of econometric estimation, which is absent in the works based on the life cycle hypothesis. In addition, we have attempted to account for the two empirical limits of the permanent income model: (a) excess smoothness of consumption relative to permanent income and (b) liquidity constraint.

To account for the excess smoothness of consumption, we have reconsidered the assumption of a time separable utility function and introduced in the model habit formation. By including habits in the consumption function, we have got reasonable parameters, as shown especially by our approximation of the degree of risk aversion of the consumers. As a consequence, the consumption model with habits implies a large degree of smoothness of consumption, i.e. the inertia of the consumption process. In a macro-econometric model, accounting for this inertia is likely to replicate the usually observed slow response of consumption to shocks and to avoid the large, unrealistic volatility of consumption that traditional Euler equation can produce.

To account for liquidity constraints, we have assumed two different types of households whose proportion in the economy is constant over time. The households of the first group are liquidity-constrained whereas the households of the second group have a free access to financial markets and behave according to the arbitrage equation. To estimate econometrically the share of the liquidity-constrained agents, we have included it directly in the Euler equation by assuming that the unconstrained households know this share and account for it in the optimisation.

The results obtained give us reasonable values for the consumption function of Marmotte. The share of liquidity constrained households is in line with recent studies on this topic. The presence of habits in the consumption decisions is empirically verified, hence supporting the specification choice. Finally, the combination of the parameters is consistent with reasonable consumers' preferences and the properties of the consumption function are likely to produce responses to shocks that are consistent with those observed.

The estimations on a panel of countries has allowed us to get both more data to make our empirical evidence more robust and to study what are the sources of structural differences across countries. By estimating the "deep" parameters of the consumption function (degree of risk aversion, degree of inertia in the consumption process, presence of habits in the consumers' preference, …), we have provided an evidence of the roots of differences. Only habit parameter seems to differ across countries. This implies some slight differences in terms of degree of smoothness of consumption. However, the main result is that differences across the 17 countries present in Marmotte are not large enough to imply significant differences in terms of consumption responses to shocks in the simulations of the model.

Appendix: estimation results

Model 1					
curvature (γ)	*Value*	*t-Stat*	**Habit (α)**	*Value*	*t-Stat*
Austria	>100	0.0	Austria	-0.77	-0.6
Belgium	>100	0.0	Belgium	-0.73	-1.0
Canada	>100	0.0	Canada	-0.70	0.0
Denmark	>100	0.0	Denmark	-0.74	-0.9
Finland	>100	0.0	Finland	-0.75	-0.2
France	>100	0.0	France	-0.76	-0.3
Germany	>100	0.0	Germany	-0.77	-2.0
Greece	>100	0.0	Greece	-0.70	-0.2
Italy	>100	0.0	Italy	-0.71	-0.3
Ireland	>100	0.0	Ireland	-0.65	-0.2
Japan	>100	0.0	Japan	-0.86	-0.1
Netherlands	>100	0.0	Netherlands	-0.72	0.0
Portugal	>100	0.0	Portugal	-0.79	-0.3
Spain	>100	0.0	Spain	-0.75	-0.1
Sweden	>100	0.0	Sweden	-0.67	-0.4
UK	>100	0.0	UK	-0.76	-0.1
US	>100	0.0	US	-0.73	-1.3
liquidity c. (λ)	*Value*	*t-Stat*	**Discount f. (β)**	*Value*	*t-Stat*
Austria	>100	0.0	Austria	0.90	0.7
Belgium	>100	0.0	Belgium	0.94	7.0
Canada	>100	0.0	Canada	0.88	0.3
Denmark	>100	0.0	Denmark	0.89	0.8
Finland	>100	0.0	Finland	0.97	0.5
France	>100	0.0	France	0.91	0.3
Germany	>100	0.0	Germany	1.14	0.4
Greece	>100	0.0	Greece	0.99	0.7
Italy	>100	0.0	Italy	0.91	0.7
Ireland	>100	0.0	Ireland	0.98	2.8
Japan	>100	0.0	Japan	0.86	0.1
Netherlands	>100	0.0	Netherlands	0.92	0.1
Portugal	>100	0.0	Portugal	0.75	0.2
Spain	>100	0.0	Spain	0.89	0.2
Sweden	>100	0.0	Sweden	0.91	1.8
UK	>100	0.0	UK	0.93	0.4
US	>100	0.0	US	0.93	0.5

Model 2			Model 3		
Curvature (γ)	*Value*	*t-Stat*	**Curvature (γ)**	*Value*	*t-Stat*
17 countries	>100	0.0	Austria	>100	0.0
habit (α)	*Value*	*t-Stat*	Belgium	>100	0.0
Austria	-0.82	-0.8	Canada	>100	0.0
Belgium	-0.76	-2.6	Denmark	>100	0.0
Canada	-0.74	-0.2	Finland	>100	0.0
Denmark	-0.79	-1.5	France	>100	0.0
Finland	-0.78	-0.4	Germany	>100	0.0
France	-0.81	-0.7	Greece	>100	0.0
Germany	-0.79	-2.4	Italy	>100	0.0
Greece	-0.70	-1.3	Ireland	>100	0.0
Italy	-0.74	-0.5	Japan	>100	0.0
Ireland	-0.64	-0.5	Netherlands	>100	0.0
Japan	-0.92	-0.1	Portugal	>100	0.0
Netherlands	-0.76	-0.1	Spain	>100	0.0
Portugal	-0.86	-1.1	Sweden	>100	0.0
Spain	-0.80	-0.7	UK	>100	0.0
Sweden	-0.70	-0.7	US	>100	0.0
UK	-0.81	-0.2	**habit (α)**	*Value*	*t-Stat*
US	-0.76	-3.2	17 countries	-0.79	-1.6
liqu. c. (λ)	*Value*	*t-Stat*	**liqu. c. (λ)**	*Value*	*t-Stat*
Austria	79.67	0.0	Austria	>100	0.0
Belgium	2.85	0.1	Belgium	5.29	0.0
Canada	>100	0.0	Canada	>100	0.0
Denmark	>100	0.0	Denmark	>100	0.0
Finland	>100	0.0	Finland	>100	0.0
France	>100	0.0	France	>100	0.0
Germany	2.06	0.1	Germany	>100	0.0
Greece	>100	0.0	Greece	>100	0.0
Italy	>100	0.0	Italy	>100	0.0
Ireland	>100	0.0	Ireland	>100	0.0
Japan	>100	0.0	Japan	>100	0.0
Netherlands	>100	0.0	Netherlands	>100	0.0
Portugal	>100	0.0	Portugal	>100	0.0
Spain	>100	0.0	Spain	>100	0.0
Sweden	>100	0.0	Sweden	>100	0.0
UK	>100	0.0	UK	>100	0.0
US	27.44	0.0	US	>100	0.0

disc. f. (b)	*Value*	*t-Stat*	**disc. f. (b)**	*Value*	*t-Stat*
Austria	0.90	1.0	Austria	0.89	1.2
Belgium	0.95	10.0	Belgium	0.91	6.0
Canada	0.90	0.5	Canada	0.80	0.3
Denmark	0.91	2.3	Denmark	0.85	0.5
Finland	0.96	1.4	Finland	0.96	1.1
France	0.91	1.4	France	0.90	1.1
Germany	1.17	1.6	Germany	1.16	0.6
Greece	0.99	9.0	Greece	0.95	3.5
Italy	0.92	1.2	Italy	0.86	1.3
Ireland	0.98	16.7	Ireland	0.94	3.0
Japan	0.83	0.2	Japan	0.94	0.1
Netherlands	0.93	0.3	Netherlands	0.87	0.5
Portugal	0.72	0.3	Portugal	0.75	0.3
Spain	0.91	1.6	Spain	0.86	0.7
Sweden	0.94	3.3	Sweden	0.82	0.9
UK	0.95	0.9	UK	0.91	0.6
US	0.94	5.9	US	0.89	0.3

Model 4			Model 5		
Curvature (γ)	**Value**	*t-Stat*	**Curvature (γ)**	*Value*	*t-Stat*
Austria	8.77	0.2	Austria	>100	0.0
Belgium	>100	0.1	Belgium	>100	0.0
Canada	>100	0.0	Canada	>100	0.0
Denmark	>100	0.0	Denmark	>100	0.0
Finland	>100	0.0	Finland	>100	0.0
France	>100	0.0	France	>100	0.0
Germany	>100	0.0	Germany	>100	0.0
Greece	>100	0.0	Greece	>100	0.0
Italy	>100	0.0	Italy	>100	0.0
Ireland	>100	0.0	Ireland	>100	0.0
Japan	>100	0.0	Japan	>100	0.0
Netherlands	>100	0.0	Netherlands	>100	0.0
Portugal	>100	0.0	Portugal	>100	0.0
Spain	>100	0.0	Spain	>100	0.0
Sweden	>100	0.0	Sweden	>100	0.0
UK	>100	0.0	UK	>100	0.0
US	50.80	0.0	US	>100	0.0
habit (α)	*Value*	*t-Stat*	**habit (α)**	*Value*	*t-Stat*
Austria	-0.83	-2.4	Austria	-0.71	-1.8
Belgium	-0.77	-2.8	Belgium	-0.71	-1.9
Canada	-0.74	-0.2	Canada	-0.52	-0.5
Denmark	-0.78	-1.0	Denmark	-0.65	-1.4
Finland	-0.77	-0.5	Finland	-0.86	-0.7
France	-0.82	-0.6	France	-0.68	-1.0
Germany	-0.78	-0.4	Germany	-0.91	-1.7
Greece	-0.70	-0.6	Greece	-0.82	-0.6
Italy	-0.74	-0.3	Italy	-0.65	-0.5
Ireland	-0.64	-0.3	Ireland	-0.77	-1.3
Japan	-0.91	-0.1	Japan	-0.78	-0.2
Netherlands	-0.76	-0.2	Netherlands	-0.65	-0.3
Portugal	-0.85	-0.4	Portugal	-0.56	-1.0
Spain	-0.80	-0.7	Spain	-0.67	-0.7
Sweden	-0.70	-0.7	Sweden	-0.59	-0.5
UK	-0.81	-0.4	UK	-0.76	-0.1
US	-0.77	-1.8	US	-0.71	-2.3

liqu. c. (l)	*Value*	*t-Stat*
17 countries	0.21	0.8
disc. f. (b)	*Value*	*t-Stat*
Austria	0.91	3.2
Belgium	0.96	8.6
Canada	0.91	0.3
Denmark	0.91	2.3
Finland	0.99	1.6
France	0.92	1.4
Germany	1.18	0.2
Greece	1.01	4.7
Italy	0.92	0.6
Ireland	0.99	7.8
Japan	0.85	0.1
Netherlands	0.93	0.6
Portugal	0.69	0.6
Spain	0.91	0.7
Sweden	0.94	3.3
UK	0.96	0.7
US	0.95	5.1

liqu. c. (l)	*Value*	*t-Stat*
Austria	>100	0.0
Belgium	18.70	0.0
Canada	>100	0.0
Denmark	>100	0.0
Finland	1.08	0.2
France	>100	0.0
Germany	0.47	0.5
Greece	>100	0.0
Italy	>100	0.0
Ireland	>100	0.0
Japan	>100	0.0
Netherlands	>100	0.0
Portugal	>100	0.0
Spain	>100	0.0
Sweden	>100	0.0
UK	>100	0.0
US	41.37	0.0
disc. f. (b)	*Value*	*t-Stat*
17 countries	0.94	9.5

Model 6			Model 7			Model 8		
curv. (g)	*Value*	*t-Stat*	**curv. (g)**	*Value*	*t-Stat*	**curv. (g)**	*Value*	*t-Stat*
17 countries	5.47	0.2	17 countries	2.72	0.4	17 countries	0.59	0.5
habit (a)	*Value*	*t-Stat*	**habit (a)**	*Value*	*t-Stat*	**habit (a)**	*Value*	*t-Stat*
17 countries	-0.80	-4.6	Austria	-0.84	-3.9	Austria	-0.70	-4.1
liqu. C. (l)	*Value*	*t-Stat*	Belgium	-0.78	-3.8	Belgium	-0.75	-2.1
Austria	27.64	0.0	Canada	-0.75	-0.5	Canada	-0.23	-0.4
Belgium	0.29	1.0	Denmark	-0.81	-1.8	Denmark	-0.92	-2.3
Canada	>100	0.0	Finland	-0.78	-1.0	Finland	-0.90	-2.3
Denmark	>100	0.0	France	-0.83	-0.9	France	-0.53	-2.4
Finland	>100	0.0	Germany	-0.81	-2.1	Germany	-0.59	-2.6
France	12.33	0.0	Greece	-0.70	-2.0	Greece	-0.83	-2.4
Germany	0.67	0.3	Italy	-0.75	-0.5	Italy	-0.61	-4.0
Greece	1.67	0.1	Ireland	-0.65	-0.9	Ireland	-0.72	-1.4
Italy	>100	0.0	Japan	-0.94	-2.9	Japan	-0.95	-1.5
Ireland	>100	0.0	Netherlands	-0.77	-0.6	Netherlands	-0.52	-1.8
Japan	>100	0.0	Portugal	-0.88	-2.1	Portugal	-0.70	-3.7
Netherlands	>100	0.0	Spain	-0.81	-1.2	Spain	-0.84	-2.6
Portugal	>100	0.0	Sweden	-0.71	-1.1	Sweden	-0.56	-1.7
Spain	6.43	0.0	UK	-0.82	-0.8	UK	-0.78	-1.0
Sweden	>100	0.0	US	-0.78	-2.5	US	-0.82	-5.7
UK	>100	0.0	**liqu. c. (l)**	*Value*	*t-Stat*	**liqu. c. (l)**	*Value*	*t-Stat*
US	1.30	0.2	17 countries	0.18	1.2	Austria	1.96	0.1
disc. F. (b)	*Value*	*t-Stat*	**disc. f. (b)**	*Value*	*t-Stat*	Belgium	>100	0.0
Austria	0.91	2.2	Austria	0.92	6.0	Canada	>100	0.0
Belgium	0.94	7.7	Belgium	0.96	14.6	Denmark	>100	0.0
Canada	0.85	0.4	Canada	0.92	1.6	Finland	10.79	0.0
Denmark	0.89	1.9	Denmark	0.92	3.1	France	>100	0.0
Finland	0.95	2.4	Finland	0.97	11.5	Germany	2.19	0.1
France	0.91	4.0	France	0.92	2.4	Greece	55.46	0.0
Germany	1.17	3.4	Germany	1.16	1.1	Italy	1.83	0.1
Greece	0.97	5.9	Greece	1.00	18.2	Ireland	>100	0.0
Italy	0.88	2.4	Italy	0.93	1.6	Japan	3.66	0.0
Ireland	0.96	6.1	Ireland	0.98	16.7	Netherlands	>100	0.0
Japan	0.97	2.3	Japan	0.85	0.9	Portugal	>100	0.0
Netherlands	0.90	1.8	Netherlands	0.94	2.8	Spain	74.21	0.0
Portugal	0.81	0.8	Portugal	0.71	0.6	Sweden	>100	0.0
Spain	0.90	2.2	Spain	0.93	2.5	UK	>100	0.0
Sweden	0.87	1.4	Sweden	0.95	7.2	US	0.41	0.5
UK	0.95	1.7	UK	0.97	4.0	**disc. f. (b)**	*Value*	*t-Stat*
US	0.92	4.1	US	0.96	9.0	17 countries	0.96	70.2

Model 9			Model 10			Model 11		
Curvature (g)	*Value*	*t-Stat*	**curvature (g)**	*Value*	*t-Stat*	**Curvature (g)**	*Value*	*t-Stat*
Austria	>100	0.0	Austria	>100	0.0	Austria	12.82	0.1
Belgium	17.62	0.1	Belgium	>100	0.0	Belgium	26.51	0.1
Canada	>100	0.0	Canada	>100	0.0	Canada	7.72	0.2
Denmark	>100	0.0	Denmark	60.79	0.0	Denmark	12.85	0.1
Finland	>100	0.0	Finland	>100	0.0	Finland	9.06	0.2
France	>100	0.0	France	>100	0.0	France	8.85	0.2
Germany	7.51	0.2	Germany	>100	0.0	Germany	1.75	0.7
Greece	12.57	0.1	Greece	>100	0.0	Greece	>100	0.0
Italy	>100	0.0	Italy	>100	0.0	Italy	>100	0.0
Ireland	35.76	0.0	Ireland	>100	0.0	Ireland	6.42	0.2
Japan	>100	0.0	Japan	>100	0.0	Japan	>100	0.0
Netherlands	>100	0.0	Netherlands	>100	0.0	Netherlands	13.30	0.1
Portugal	>100	0.0	Portugal	>100	0.0	Portugal	>100	0.0
Spain	>100	0.0	Spain	>100	0.0	Spain	>100	0.0
Sweden	>100	0.0	Sweden	>100	0.0	Sweden	>100	0.0
UK	>100	0.0	UK	>100	0.0	UK	>100	0.0
US	24.45	0.1	US	>100	0.0	US	2.37	0.8
Habit (a)	**Valu**	*t-Stat*	**habit (a)**	**Valu**	*t-Stat*	**habit (a)**	**Valu**	*t-Stat*
17 countries	-0.78	-4.0	17 countries	-0.69	-4.8	Austria	-0.72	-6.2
Liquity c. (l)	*Value*	*t-Stat*	**liquity c. (l)**	*Value*	*t-Stat*	Belgium	-0.81	-2.6
17 countries	0.41	0.6	Austria	0.65	0.3	Canada	-0.34	-0.7
Disc. f. (b)	*Value*	*t-Stat*	Belgium	0.78	0.3	Denmark	-0.73	-1.7
Austria	0.89	1.9	Canada	>100	0.0	Finland	-0.72	-0.8
Belgium	0.91	7.1	Denmark	>100	0.0	France	-0.62	-2.2
Canada	0.81	0.3	Finland	0.88	0.3	Germany	-0.64	-2.9
Denmark	0.84	1.3	France	0.62	0.2	Greece	-0.88	-0.2
Finland	0.96	4.0	Germany	>100	0.0	Italy	-0.61	-1.5
France	0.90	4.1	Greece	4.07	0.1	Ireland	-0.72	-5.5
Germany	1.10	3.4	Italy	>100	0.0	Japan	-0.92	-1.7
Greece	0.97	5.3	Ireland	>100	0.0	Netherlands	-0.57	-1.5
Italy	0.86	1.6	Japan	0.62	0.4	Portugal	-0.50	-1.1
Ireland	0.94	3.9	Netherlands	34.08	0.0	Spain	-0.69	-4.2
Japan	0.94	2.8	Portugal	>100	0.0	Sweden	-0.48	-1.0
Netherlands	0.87	1.5	Spain	>100	0.0	UK	-0.87	-0.4

Portugal	0.77	0.4	Sweden	>100	0.0	US	-0.82	-6.8
Spain	0.87	0.9	UK	>100	0.0	**liquity c. (l)**	*Value*	*t-Stat*
Sweden	0.83	1.0	US	1.26	0.1	17 countries	0.09	1.7
UK	0.92	0.7	**disc. f. (b)**	*Value*	*t-Stat*	**disc. f. (b)**	*Value*	*t-Stat*
US	0.89	3.2	17 countries	0.96	20.1	17 countries	0.97	27.9

Model 12		
curv. (γ)	*Value*	*t-Stat*
17 countries	1.62	0.7
habit (α)	**Value**	*t-Stat*
17 countries	-0.81	-5.8
liquity c. (λ)	*Value*	*t-Stat*
17 countries	0.18	1.4
disc. f. (β)	*Value*	*t-Stat*
Austria	0.94	12.1
Belgium	0.96	10.8
Canada	0.90	12.3
Denmark	0.92	7.2
Finland	0.97	12.2
France	0.93	19.8
Germany	1.17	4.0
Greece	0.99	8.8
Italy	0.90	8.2
Ireland	0.97	8.8
Japan	0.99	10.7
Netherlands	0.93	12.8
Portugal	0.84	7.5
Spain	0.93	10.2
Sweden	0.91	11.9
UK	0.98	10.1
US	0.95	9.9

Model 13		
Curv. (γ)	*Value*	*t-Stat*
17 countries	>100	0
Habit (α)	**Valu**	*t-Stat*
17 countries	-0.92	-29.7
Liquity c. (λ)	*Value*	*t-Stat*
Austria	13.04	0.0
Belgium	1.07	0.2
Canada	>100	0.0
Denmark	2.31	0.1
Finland	7.33	0.0
France	1.20	0.1
Germany	0.94	0.1
Greece	>100	0.0
Italy	>100	0.0
Ireland	1.61	0.1
Japan	>100	0.0
Netherlands	>100	0.0
Portugal	>100	0.0
Spain	0.65	0.4
Sweden	>100	0.0
UK	1.44	0.2
US	0.82	0.3
Disc. f. (β)	*Value*	*t-Stat*
17 countries	0.96	183.7

Model 14		
curv. (γ)	*Value*	*t-Stat*
17 countries	0.84	1.1
habit (α)	**Value**	*t-Stat*
Austria	-0.74	-8.7
Belgium	-0.64	-3.1
Canada	-0.20	-0.4
Denmark	-0.83	-2.7
Finland	-0.63	-1.7
France	-0.70	-4.8
Germany	-0.63	-3.4
Greece	-0.94	-21.9
Italy	-0.64	-4.2
Ireland	-0.82	-6.0
Japan	-0.97	-13.5
Netherlands	-0.63	-2.8
Portugal	-0.69	-6.1
Spain	-0.73	-6.6
Sweden	-0.39	-1.4
UK	-0.82	-1.5
US	-0.81	-5.1
Liquity c. (λ)	*Value*	*t-Stat*
17 countries	0.13	1.6
disc. f. (β)	*Value*	*t-Stat*
17 countries	0.96	75.0

Model 15		
curv. (γ)	*Value*	*t-Stat*
Austria	2.16	0.9
Belgium	5.74	0.3
Canada	>100	0.0
Denmark	5.90	0.2
Finland	2.38	0.7
France	3.03	0.6
Germany	>100	0.0
Greece	0.93	0.6
Italy	1.63	1.1
Ireland	8.57	0.1
Japan	0.94	1.1
Netherlands	3.37	0.4
Portugal	>100	0.0
Spain	2.17	1.0
Sweden	33.62	0.0
UK	1.98	0.6
US	1.77	1.1
habit (α)	**Value**	*t-Stat*
17 countries	-0.75	-14.4
liquity c. (λ)	*Value*	*t-Stat*
17 countries	>100	0
disc. f. (β)	*Value*	*t-Stat*
17 countries	0.96	44.3

Model 16		
Curvature (γ)	*Value*	*t-Stat*
17 countries	0.03	1.6
Habit (α)	**Value**	*t-Stat*
17 countries	-0.92	-158.8

Liquidity c. (l)	*Value*	*t-Stat*
17 countries	9.72	0.0
Discount f. (b)	*Value*	*t-Stat*
17 countries	0.96	329.3

CHAPTER III

ACCOUNTING IDENTITIES, WAGE CURVE, FOREIGN TRADE AND SPECIFIATION OF POLICY RULES

I ACCOUNTING IDENTITIES

Marmotte is a one good model except for trade where there is a distinction between primary commodities and the rest of traded goods. Hence there is one composite good for all goods and services traded in the domestic economy. This implies that this composite good is the same for production and investment. The economy is based on two distinguished agents: the government and the private sector.

I.1 Goods and services equilibrium

Equations (1) to (3) are traditional accounting identities. The left-hand side of equation (1) represents GDP at market price (*pmp*). The right-hand side sums the various components of demand. Households' consumption is valued at consumption price, *pc*. Government consumption (*G*) and private and government investments (*J* and *Jg*) are valued at the absorption price *pa* (the deflator of *G+JG+J*, indexed on *pc* net of Vat[22]). Exports and imports of non-primary goods and services (*EXMA* and *IMM*) and the net exports of commodities (*NXCA*) are valued at their respective prices, *pxm*, *pmma*, *pco*. The price of primary commodities (*pco*), measured in dollar, is converted in national currency using the nominal exchange rate (*e*). All price indices are based in year 1990. Government's consumption is split into consumption in public services, measured by the labour cost of civil servants (with real wage rate *wgrr* and employment *Eg*), and the consumption of private goods and services *G*. *sdvr* and *sqd* represent errors and omissions (negligible for most of the countries).

$$(1)\quad pmp_t.GDP_t = pc_t.C_t + pa_t(G_t + J_t + Jg_t) + (pc_t.Eg_t wgrr_t) + pxm_t.EXMA_t - pmma_t.IMM_t + pco_t NXCA_t.e_t / e90 + pmp_t.sdvr_t$$

The value added tax receipts, denoted *vat* is defined as follows:

$$(2)\quad vat_t = pc_t.C_t.vatr_t / (vatr_t + 1)$$

$$(3)\quad GDP_t = C_t + G_t + (pc_t.Eg_t wgrr_t) / pmp_t + Jg_t + J_t + EXMA_t - IMM_t + NXCA_t + sdq_t$$

Marmotte assumes some stickiness of production prices. In each country, the rate of variation of this price is proportional to the log of the ratio between the demand and the supply of goods produced by the national private sector. Current production is assumed to be equal to demand. Supply determines potential production. In the long run, effective and potential outputs are equal. This stickiness of prices is introduced to give Marmotte some Keynesian features in the short run.

[22] The value-added tax is supposed to be only based on households' consumption.

(4) $$\ln\left(\frac{pmp_t.GDP_t - pc_t Eg_t wgrr_t - vat_t - pmp_t.vad_t}{p_t YQ_t}\right) = \frac{1}{\lambda}\ln\left(\frac{p_t}{p_{t-1}.\pi^*}\right)$$

where *vad* is the value added discrepancy in constant price, *YQ* is the production of total industries, *p* is the production price and π^* is the steady state level of the inflation rate.

I.2 Domestic prices

Consumer price index, vat non-included, is a geometric average of GDP deflator at factor costs, non primary good and service import price and commodity price.

(5) $$\ln(pc_t) = \alpha_1 \ln(p_t) + (1-\alpha_1)\left[\alpha_2 \ln(pmma_t) + (1-\alpha_2)\ln(pco_t.e_t/e90)\right] + \ln\left(\frac{1+vatr}{1+vatr90}\right)$$

Absorption price (pa_t) and consumer price (pc_t) are both equal to 1 in the base year. They may differ later, either because the value added tax rate is modified or for other reasons such as a modification of the respective share of national goods in consumption and investment and Government spending.

(6) $$pa_t = pc_t \frac{1+rpc}{1+vatr}$$

where *rpc* is a relative price correction between *pa* and *pc*, not taken into account by the value added tax evolution.

I.3 Government

The tax revenue of Government (*TT*) is composed of value added tax (*Vat*), social security contributions (with rate *sccr*), corporate taxes (with rate *tcr*), income tax (with rate *tlr*) and other taxes (with rate *otr*). Other taxes are used as a device to stabilise public debt and prevent Government from entering a *Ponzi* finance process. They are modelled as a fiscal policy rule that implies an increase in these taxes when public debt is larger than the target set by the Government.

(7) $$\begin{aligned} TT_t &= Vat_t + sccr.(p_t.wprr_t.Ep_t + pc_t wgrr_t.Eg_t) + (1-sccr).tlr. \\ &(p_t.wprr_t.Ep_t + pc_t wgrr_t.Eg_t) + tcr.(p_t YQ_t - p_t wprr_t Ep_t) + otr_t.pmp_t GDP_t \end{aligned}$$

where *Ep* is employment in the private sector.

The fiscal rule is included in the model for technical reason, but is also a representation of how an instrument of fiscal policy reacts to the state of the economy and thus is part of the overall fiscal policy. The rate of other taxes is thus dependent on the ratio of the debt to the nominal GDP, defined as:

(8) $$otr_t = \beta_0 \frac{D_t}{pmp_t.GDP_t} + \beta_1$$

The current spending of government sums its consumption and investment in private goods, the labour cost of civil servants, transfers *Trr* (including the benefits to the unemployed), and the interest paid on public debt. Finally, the last part of the government surplus represents the other net receipt of government (*Ondgr*) in constant price.

(9)
$$Pb_t = TT_t - (G_t + Jg_t).pa_t - Eg_t\, pc_t.wgrr_t - pmp_t.Trr_t$$
$$-[(1-bilg_t).i_t + bilg_t i_t^{LT}].D_t + pmp_t Ondgr_t$$

where Onr_t are other net receipts of government in constant price, *Trr* are the net general government transfers in constant prices.

Government debt at the beginning of the year is equal to the Government debt at the beginning of the previous year plus the government surplus of the previous year, and the capital gain of the holders of perpetual bonds made over the previous year. D_t represents government debt at the beginning of the year *t*, made up of short-term bills accounting for a fraction (1-*bilg*) of total debt and of perpetual bonds yielding each a yearly payment of 1 currency unit. The price of one bond is denoted $1/i_t^{LT}$, with i_t^{LT} defined as the long run interest rate.

At the beginning of the previous year, the value of the long-term debt was equal to $bi\lg_{t-1}.D_{t-1}$, representing $i_{t-1}^{LT}.bilg_{t-1}.D_{t-1}/i_t^{LT}$ bonds. Its value at the beginning of the current year is thus $i_{t-1}^{LT}.bi\lg_{t-1}.D_{t-1}/i_t^{LT}$.

(10) $$D_t = D_{t-1} - Pb_{t-1} + bilg_{t-1}\left(\frac{i_{t-1}^{LT}}{i_t^{LT}} - 1\right)D_{t-1}$$

I.4 Balance of Payment

(11) $$TB_t = pxm_t.EXMA_t - pmma_t.IMM_t + pco_t.e_t / e90.NXCA_t$$

Net foreign assets are held in perpetual bonds denominated in US dollars with a fraction (*bile*) of total net foreign assets and in short term bills accounting for a fraction (1-*bile*). These assets are supposed to pay respectively the US short term interest rate (i_t^{US}) and the long term interest rate ($i_t^{LT,US}$). Nfa_t represents the net foreign assets at the beginning of the period. These assets are equal to the sum of their value at the beginning of the previous year, the current account surplus of the previous year (trade balance *TB* + net transfers *CBTR* + interest payments), and the capital gain of the holders of perpetual bonds made over the previous year.

(12) $$Nfa_t = \frac{TB_{t-1} + pmp_{t-1}CTBR_{t-1}}{e_{t-1}} + Nfa_{t-1}\left[(1-bile_{t-1})(1+i_{t-1}^{US}) + bile_{t-1}\frac{i_{t-1}^{LT,US}}{i_t^{LT,US}}\right]$$

where $bile_t$ is the share of long term external debt and $CTBR_t$ is the current transfer balance, in domestic currency and constant price, which is exogenous.

I.5 Employment

Total employment is composed of private (*Ep*) and public employment (*Eg*).

(13) $$ET_t = Ep_t + Eg_t$$

II THE WAGE CURVE[23]

This section offers an estimation of private wage behaviour on a panel of 16 industrialised countries of Marmotte with yearly data. Using a panel estimation helps us to get more robust and precise empirical findings: as these countries share some common structural features, each country estimation benefits from information brought by its 15 partners. Second, panel estimation allows us to identify deep structural differences between countries. This kind of analysis is particularly important as industrialised countries' labour markets display great heterogeneity concerning wage bargaining processes, degrees of job protection, and provision of replacement incomes, etc. (See OECD (1994), Cadiou and Guichard (1999)).

The first paragraph proposes a simple formalisation of wages setting, based on a wage curve in which labour cost depends on labour productivity, prices, the wedge between real labour cost for firms and the purchasing power of nominal wages for wage earners, and the employment rate. We also introduce nominal rigidities in this equation: some wage contracts are longer than one year and depend not only on current prices but also on anticipated ones. Paragraph 2 is dedicated to econometric questions. The third paragraph gives the results.

II.1 The theoretical background

Each year, firms and workers are assumed to agree on nominal wage contracts. Some of them expire during the current year (their length is shorter than one year); others run out the following year (their length is shorter than two years[24]). Each contract is identified by two indices: t the year it is concluded and i which is equal to 1 if the contract expires during the current year, 2 if it expires during the following year. The theoretical models of wage setting (bargaining models, search models, efficient wage models) justify the following equation[25] :

(1) $Log(WC_{t,i} / PE_{t,i}) = w_0 + Log(Y_t / E_t) + w_2 Log(WEDG) + w_3 Log(ER_t) + e_{t,i}$

WC is the private wage cost set by the contract. PE is the average expected price level during the contract. Y is the private output. E represents private employment. ER is the employment rate (in percent) defined as the ratio between total employment and active population. *WEDG* is the wedge between real labour cost for firms and the purchasing power of nominal wages for the wages earners. It is defined as (pc/(p*(1-sscr)*(1-tlr)*(1+vatr90)) where pc is the consumer price index, sscr the social security contribution rate (for both employers and employees), tlr the tax rate on labour income, vatr90 the VAT rate for the base year.

We assume wage cost to be perfectly indexed on the average price expected during the contract, and on the current productivity of labour[26]. The error term e includes all

[23] This section is an adaptation of Guichard and Laffargue (2001).

[24] However these contracts might also be quite short, for instance a contract concluded at the end of a given year and expiring at the beginning of the following year.

[25] See, for instance, Layard, Nickel and Jackman (1991), Blanchflower and Oswald (1994), Pissarides (1998).

[26] This last assumption is rejected over the period of estimation. Actually the coefficient of the productivity term is around 0.7. This reflects the fact that for most countries the share of wages in output has steadily decreased over the estimation period. We argue in this volume that the putty clay side of the model gives a good explanation of this trend and shows that it is

unobservable variables influencing the wage bargaining process. They include the bargaining power of unions, the market power of firms, social laws that modify the power of insiders (for instance layoff rules), unemployment benefits, leisure utility, etc. These unobservable variables probably exhibit great persistence.

We consider two expected prices. The first is associated with contracts running out during the current year and is the current price of private output.

(2) $\quad Log(PE_{t,1}) = Log(P_t)$.

The second is associated with contracts that will expire during the following year; and is equal to the price expected for the following year.

(3) $\quad Log(PE_{t,2}) = Log({}_tP^a_{t+1})$.

Lets call p the proportion of wage contracts expiring during the year they have been concluded and W_t the average wage cost during this year. We then have the identity:

(4) $Log(W_t) = [(1-p)Log(WC_{t-1,2}) + pLog(WC_{t,1}) + (1-p)Log(WC_{t,2})]/(2-p)$ As WC and PE are not observable, we should eliminate them from this equation. Using (1) to (4), we get:

(5) $Log(W_t) =$

$$\{\ (1-p)[Log({}_{t-1}P^a_t) + w_0 + Log(Y_{t-1}/E_{t-1}) + w_2Log(WEDG_{t-1}) + w_3Log(ER_{t-1})) + e_{t-1,2}]$$
$$+ p[Log(P_t) + w_0 + Log(Y_t/E_t) + w_2Log(WEDG_t) + w_3Log(ER_t) + e_{t,1}]$$
$$+ (1-p)[Log({}_tP^a_{t+1}) + w_0 + Log(Y_t/E_t) + w_2Log(WEDG_t) + w_3Log(ER_t) + e_{t,2}]\ \}/(2-p)$$

With $(1-p)/(2-p) = r$ and $p/(2-p) = 1-2r$, (5) becomes:

$$
(6)\quad
\begin{aligned}
&Log(W_t) = w_0 + [rLog(Y_{t-1}/E_{t-1}) + (1-r)Log(Y_t/E_t)] \\
&+ w_2[rLog(WEDG_t) + (1-r)\log(WEDG_{t-1})] + w_3[rLog(ER_t) + (1-r)\log(ER_{t-1})] \\
&+ rLog({}_{t-1}P^a_t) + (1-2r)Log(P_t) + rLog({}_tP^a_{t+1}) \\
&+ re_{t-1,1} + (1-2r)e_{t,1} + re_{t,2}
\end{aligned}
$$

In the simulations of Marmotte we introduce the expected and observed rates of inflation :

$INFLE_t = {}_tP^a_{t+1}/P_t$, $INFL_t = P_t/P_{t-1}$

Then, we write equation (6) as :

---

transitory. So, this coefficient is probably biased in our estimation of the wage curve for reasons which are not entirely clear to us. So we have preferred to constraint this coefficient to 1.

$$
(7)\quad
\begin{aligned}
&Log(W_t / P_t) = w_0 + [rLog(Y_{t-1} / E_{t-1}) + (1-r)Log(Y_t / E_t)] \\
&+ w_2[rLog(WEDG_t) + (1-r)\log(WEDG_{t-1})] + w_3[rLog(ER_t) + (1-r)\log(ER_{t-1})] \\
&+ rLog(INFLE_{t-1} / INFL_t) + rLog(INFLE_t)
\end{aligned}
$$

To estimate equation (6) we must eliminate the expected variables which are not observable. We define as $h_{t-1,t}$ the forecast error made at $t-1$ for price level in t ($h_{t-1,t} = Log(P_t) - Log({}_{t-1}P_t^a)$).Under the assumption of rational expectation, this error follows a martingale difference. Then (6) becomes:

$$
(8)\quad
\begin{aligned}
&Log(W_t) = w_0 + [rLog(Y_{t-1}/E_{t-1}) + (1-r)Log(Y_t/E_t)] \\
&+ w_2[rLog(WEDG_t) + (1-r)\log(WEDG_{t-1})] + w_3[rLog(ER_t) + (1-r)\log(ER_{t-1})] \\
&+ (1-r)Log(P_t) + rLog(P_{t+1}) \\
&+ r(-h_{t-1,t} + h_{t,t+1}) + re_{t-1,1} + (1-2r)e_{t,1} + re_{t,2}
\end{aligned}
$$

Hence, nominal wage cost depends on a weighted average (with the same weights) of current and past productivities and employment rates. It also depends on future and current prices.

We mentioned previously that it is likely for the error terms $e_{t,1}$ and $e_{t,2}$ to be highly persistent. So, we assume that they can be represented by an AR(1) process of parameter r. We tried to estimate this parameter and always got a value very close to 1. Therefore, we decided to assume that the error term follows an integrated process of order 1. That means that the unobservable variables are I(1), which is quite a reasonable assumption. This implies that the observable variables of equation (8) are not cointegrated. However, the values of the elasticities in this equation are of great interest for the economist. We then estimate the first difference of (8), which also allows us to eliminate a constant that is very likely to vary across countries[27].

II.2 Econometric methodology

Our aim is to estimate the first difference of (8) on a panel of countries. The specification of this wage equation is the same for all countries, but the values of the parameters may differ.

The sample of 16 industrialised countries includes Germany, Austria, Belgium, Canada, Spain, the United States, Finland, France, Greece, Italy, Ireland, Japan, the Netherlands, Portugal, the United Kingdom, Sweden[28]. The data are annual[29], taken from OECD Economic Outlook and expressed in logarithms.

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[27] In the new equation, the error term is an ARMA of probably low order. As said in the second section we could not retain this in the estimation, but the tests suggest this is not an important problem.

[28] Two countries of the EU are missing (Denmark and Luxembourg) because the data were not available. The estimation method, defined in section 2, has been implemented on TSP 4.4. The program, with detailed comments, is available upon request.

[29] The main reason for choosing yearly data results from the non availability of quarterly data for many countries of our sample. Even if quarterly data bring information on short term dynamics, it does not help much for the estimation of long term parameters, that are of greater

The estimation period goes from 1982 to 1997 (the same for each country). The data are available on a longer period, but we decided to start in 1982 to prevent bias due to structural breaks in the equations: European countries implemented numerous reforms in the late 70's and at the beginning of the 80's. We consider that the cross-country dimension compensates for this relatively short period. Of course, economic shocks and labour market reforms also occurred during the 80's and 90's, but it seemed unwise to shorten the period more. Dummies were added to deal with the German reunification as the data are available for the whole period neither for the Western part nor for the whole Germany. This is equivalent to excluding two years (1991 and 1992) out of the German equation. On the whole, our estimations concern 16 countries and 15 years.

Our problem is to estimate on a panel of I countries, indexed by i, and on a period of T years, indexed by t, the following system of I equations:

(9) $$y_{it} = g_i(y_{i,t-1}, x^a_{1i,t+1}, x_{2it}, x_{3it}; a_i) + e_{it}; i = 1,..,I; t = 2,..,T-1 ,$$

I and T are of the same order of magnitude, and not very high. The y_{it} are the endogenous variables, the x_{jit} are the exogenous variables, f_i is a function representing the behaviour associated to country i. x_{2it} is predetermined, but this property is not shared by variable x_{3it}. The endogeneity of x_{3it} prevents the nonlinear least squares estimators of the parameters of being consistent. Moreover, variable $x^a_{1i,t+1}$ represents the forecast at time t of variable x_{1i} for time $t+1$. The a_i are the parameters of this function. Some of these parameters are country specific, the others are common to some countries or to all of them. e_{it} is the error term of null expected value. Thus, we face the same problem as in Appendix 1, and we will use the GMM method developed in this chapter.

. The instruments we choose are the constant term, the wage cost, the productivity of labour and the employment rate, a lag of three periods. Then, we have retained four instruments per country, so a total of: 1+3*16=49 instruments. Taking a higher number of instruments would increase the number of degrees of freedom, but would raise the risk of small sample biases.

II.3 Results

The test strategy presented in Final Appendix 1, used to determine which parameters change and which parameters have the same value between countries concluded that the employment rate and the wedge parameters are country specific and the proportion of long terms contracts in the same in all nations. The results are given in Table 1.

Parameter r is equal to 0.17, meaning that 80% of wages contracts expire the year they have been concluded.

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economic interest (in our case the long run elasticity of wages to employment or wedge). Moreover, the quarterly data are often seasonally adjusted; that deteriorate the informative content of the series and is a source of bias in the estimations ant the related tests. (see Hendry (1995, p. 559-565))

The elasticity of wage cost to the wedge (w_2) is almost always very low and non significant. We have put it to 0, except for France, Netherlands and Canada. For the first and the last of these three countries the value of this coefficient is low. .This implies that an increase in social security contributions (on employers and employees) results in general in no or a small increase in wage costs. Thus, it is mainly borne by employees (whose net earnings fall). This result confirms the conclusion by Cotis and Loufir (1990) in the case of France.

The employment rate has a negative or an insignificant effect on wages in Belgium, Canada, Ireland, Japan, Portugal, Spain and the United States. So, we have constrained its coefficient to be equal to 0 for these countries. It has a significant positive effect on wages in Austria, France, Germany, Italy, and the Netherlands. It is positive but not significant for Finland, Greece, Sweden and the United Kingdom. The traditional result of a high wage flexibility in Italy is confirmed30, as the low flexibility of wages in the UK.

Usually, wage flexibility is estimated regarding the unemployment rate and not to the employment rate. Everything being equal, this elasticity is $-w_{3i}\overline{u}_i/\left(1-\overline{u}_i\right)$, with $\overline{u}_i$ the average unemployment rate for country i. We give these elasticities in the second column of Table 1. Their values are consistent with the conclusion of Blanchflower and Oswald (1995) on individual data sets (that is –0.1 in most countries)[31.] On the whole, even in these countries, where labour market tensions have a significant impact on the wage cost, labour market flexibility seems rather low.

[30] Layard, Nickell and Jackman (1991), Tyrvaïnen (1995), Mc Morrow (1996), Sinclair and Horsewood (1997), Roeger and in't Veld (1997), Cadiou, Guichard and Maurel (1999))

[31] The ranking of countries is quite different from Blanchflower and Oswald (the diversity of estimation periods in their study is however an important obstacle to national comparisons).

Table 1: The results with the employment rate

		Coefficient	Student
R		0.17	1.53
W2			
France		0.30	1.89
Netherlands		0.52	1.50
Canada		0.20	3.80
W3	$-w_{3i}\overline{u}_i$		
Austria	-0.064	1.21	1.67
Belgium	0		
Canada	0		
Finland	-0.05	0.42	5.06
France	-0.12	1.17	2.59
Germany	-0.27	3.52	3.23
Greece	-0.02	0.31	0.29
Ireland	0		
Italy	-0.15	2.15	6.17
Japan	0		
Netherlands	-0.18	2.36	3.44
Portugal	0		
Spain	0		
Sweden	-0.05	0.86	1.17
UK	-0.02	0.23	0.74
United States	0		

The pvalue of the Hansen test is equal to 2.1%.

In Spain, this effect is negative and significant. We can find some piece of explanation in the specificities of the labour market working or the economic shocks that have hit the countries. Franks (1994) estimates Spanish wage equation from 1976 and 1992 and finds a significant negative impact of productivity and a positive and significant impact of unemployment. He interprets this result as a consequence of the great rigidity of Spain's labour market and of its deep duality. The 80's were characterised by deregulation of the labour market designed to increase its flexibility, mainly through the introduction of fixed term contracts. These new contracts met a big success with employers and they are now representing 30% of employment. However, by reducing the risk for the long-term employees to lose their job, they have increased their bargaining power[32]. The important power of these insiders is an essential explanation of the fact that the degradation of employment did not result in a fall of real wages. The Spanish case is problematic: its particular institutions prevent the labour market to

[32] See Bentolila and Dolado (1994).

constitute a source of adjustment to macroeconomic shocks; moreover they may lead to very different reactions of the Spanish economy in face of a common shock in Europe.

III FOREIGN TRADE

This part presents the specification and the estimation of the foreign trade block of Marmotte. This block is traditional in its theoretical foundations. The usual framework for empirical works on foreign trade is derived from the model with imperfect substitutes whose foundations are due to Armington (1969). In an open economy, the volumes of imports and exports come from the maximisation of the consumers' utility under their budget constraint. The solution of this program makes the volumes of trade dependent on demand terms and price competitiveness indicators. This model has been improved by several authors such as Dixit and Stiglitz (1977), Helpman and Krugman (1985) or more recently Erkel-Rousse (1997). In our study, the model with imperfect substitutes enables us to explain in a multi-country framework the price gaps for a same good (or service) between two regions.

The originality of the present work rests more on the definition of foreign demand and price competitiveness and on the econometric methodology used to get the parameter values. In the first part we present the specification of the foreign trade block and the different procedures used to transform raw data and to compute required indicators. In the second part, we present the econometric results, which are consistent with the previous works. Besides, the nested test strategy put forward the existence of structural differences across countries, especially in terms of sensitiveness to price and demand variations.

III.1 Specification of the foreign trade block[33]

The data

In Marmotte, there is only one composite good. However, for foreign trade, primary commodities are distinguished from goods and services as a whole. Primary commodities encompass coal, petroleum, gas, cereals, other agricultural products and inedible agricultural products. This decomposition is justified first because primary commodity trade corresponds to behaviors that are specific. Besides, primary commodity markets are pretty well integrated which implies that the assumption of a unique world price is not unreasonable. Finally, distinguishing primary commodities from the rest of the goods will enable us to study the effects of shocks on the primary commodity price. This will constitute the only reasonable scenario to simulate the impact of an oil price increase on the developed economies.

To decompose trade flows (value and volume) into commodities and manufactured goods (and services), we have used three different databases: the OECD National Account database (the raw data of Marmotte), the IMF international financial statistics and the CHELEM database according the following diagram.

[33] Much of this section is based on Hervé (2001).

<table>
<tr><td>National Account data</td><td colspan="3">Total exports</td><td colspan="3">Total imports</td></tr>
<tr><td>Decomposition With IMF IFS</td><td>Exports of services</td><td colspan="2">Exports of goods</td><td colspan="2">Imports of goods</td><td>Imports of services</td></tr>
<tr><td>Decomposition With CHELEM</td><td></td><td>Exports of manufactured goods</td><td>Exports of primary commodities</td><td>Imports of primary commodities</td><td>Imports of manufactured goods</td><td></td></tr>
<tr><td>Marmotte data</td><td colspan="2">Exports of manufactured goods and services (EXM)</td><td colspan="2">Net exports of primary commodities (NXC)</td><td colspan="2">Imports of manufactured goods and services (IMM)</td></tr>
</table>

The shares calculated from the previous database are based on value data. We apply them to total exports and import in volume (National Account basis) to derive the different corresponding series in volume. Only the volume of primary commodity trade is defined by dividing the values by the unique world price. This world price index for primary commodities is defined by the IMF. The index accounts for the price of petroleum and non-fuel primary commodities (food, beverages, agricultural raw materials, metals, and fertilisers). Weights are based on 1987-89 average world export earnings. The weights are 57 percent for the index of non-fuel primary commodity prices and 43 percent for the index of petroleum prices.

a) Specification for the primary commodity trade

Net exports of primary commodities are related to domestic activity by an *ad hoc* equation. We assume that the countries of Marmotte are net importers of primary commodities. Hence, an increase in domestic activity (*ACT*) decreases the net exports of primary commodities of the country.

$NXC=-nxc1.ACT$

The world price of commodities is simply indexed to the US GDP deflator at factor cost.[34]

$$\log(PCO) = \log(P_{US})$$

b) Import equation of manufactured goods and services

According to the theory, import equations feature a national income term and a relative-price indicator, defined as the ratio between the domestic price and the import price. The import equation in Marmotte is an error correction model in which imports depend in the long run on

[34] This very simple assumption for the primary commodity trade can easily be improved for more specific needs such as the impact of an oil price increase on the developed economies.

domestic activity (*ACT*) and competitiveness (absorption price relative to import price). The specification for the import equation is as follows[35]:

$$(1)\quad \begin{aligned} Log\left(IMM_t / IMM_{t-1}\right) &= imm0 + imm1.Log\left(ACT_t / ACT_{t-1}\right) \\ &+ imm2.Log\left((pa_t / PMM_t)/(pa_{t-1}/PMM_{t-1})\right) \\ &+ imms.\begin{bmatrix} Log(IMM_{t-1}) - imm3Log(ACT_{t-1}) \\ - imm4.Log\left(pa_{t-1}/PMM_{t-1}\right) - imm5.trend \end{bmatrix} \end{aligned}$$

where *IMM* are respectively imports of goods and services in constant price 1990, denominated in domestic currency. *ACT* is the final demand also in constant price and in domestic currency. *pa* and *PMM* denote respectively the absorption price and the import prices of goods and services in domestic currency. Domestic activity is the sum of each demand component, weighted by its propensity to import:

$$(2)\quad ACT = act1.C + act2.(J + Jg) + act3.G + act4.EXM$$

with *C*, *J*, *Jg* and *G* denoting respectively households' consumption, private and public investment and public expenditures, all in constant price, 1990. *EXM* are exports of manufactured goods and services in national currency and in constant price 1990

b) import price equation

The import price (*PMM*) of a country *i* is a geometric average of the export price of each partner country (countries j) multiplied by the share of country *i*'s imports coming from country j (SM_j^i). In most of the models, the trade shares are computed for a reference year. The idea retained here is that the trade structure has modified between 1970 and 1996. To account for the structural change of market shares, we have preferred to define time varying market shares. The values of these shares for 1970 and 1996 are displayed in Table 1.

Computations[36] have been realised out of the model from the bilateral trade flows of the Chelem database. For data availability reasons, these market shares are computed for goods and not for goods and services. Besides, the bilateral trade is in nominal terms and is very volatile. Then, the market shares have been smoothed by a Hodrick-Prescott filter.

The specification of the import price equation for modelled country is as follows:

$$(3)\quad Log(PMM_i) = \sum_{\substack{j=1 \\ j \neq i}}^{n} SM_j^i Log\left(PXM_j . \frac{E_j^{90} E_i}{E_j E_i^{90}} \right)$$

[35] Sometimes, studies on the import equations include also a term measuring tensions on production capacity. However, a direct measure of the capacity utilization rate is not available in all the countries explaining then why we have excluded this variable.

[36] Since we have retained time varying market shares for the 17 countries, this represents a table with 27 (1970-1996) columns with at least 289 (17*17) lines. These market shares will not be presented here ; they are however available on request.

where *j* includes the rest of the world and with E_i the exchange rate of country *i*, E_i^{90} the exchange rate of *i* in 1990 and PXM_j denotes the export prices in national currency of country *j*.

Table 1: Direction of trade
Percent of total exports and imports

Destination Of exports	United States	Canada	Japan	Germany	France	UK	Italy	Other Europe	Rest of the world
Exports, 1970									
United States	-	27.4[a]	6.0	5.1	4.0	5.7	2.7	9.1	40.0
Canada	74.1	-	2.2	1.5	0.9	6.8	0.7	2.5	11.3
Japan	30.5	3.1	-	2.6	1.0	2.3	0.8	4.7	55.0
Germany	9.0	1.1	1.3	-	12.7	4.2	7.4	33.7	30.6
France	5.5	1.0	0.8	19.8	-	5.1	9.4	22.7	35.7
UK	11.5	3.7	1.6	5.0	4.3	-	2.3	24.9	46.7
Italy	9.8	1.1	0.7	19.6	1.4	4.5	-	16.0	34.0
Other Europe	7.2	1.0	0.7	18.2	9.5	11.1	4.1	22.1	26.1
Exports, 1996									
United States	-	22.8	9.1	4.9	3.7	5.5	1.8	8.1	44.1
Canada	80.0	-	2.9	1.5	1.0	2.1	0.7	2.1	9.7
Japan	29.6	2.3	-	4.7	1.8	3.1	0.9	5.6	52.0
Germany	7.2	0.9	1.9	-	11.1	8.4	8.1	31.4	31.0
France	6.6	1.0	1.6	16.4	-	9.2	10.2	23.6	31.4
UK	11.3	1.6	2.2	11.5	9.0	-	4.7	26.2	33.5
Italy	7.8	1.0	1.8	17.3	14.9	6.7	-	17.3	33.2
Other Europe	5.2	0.7	1.4	18.9	11.1	10.7	5.8	20.6	25.6

[a] Interpretation : 27.4% of the US exports are for the Canadian market. In Marmotte, this share corresponds to WM_{us}^{ca} (the share of country *us* exports to *ca* in *us* total export).

Origin of Imports	United States	Canada	Japan	Germany	France	UK	Italy	Other Europe	Rest of the world
Imports, 1970									
United States	-	24.7[b]	18.4	9.4	2.7	6.5	3.8	8.0	26.5
Canada	75.1	-	4.9	2.9	0.6	0.8	4.9	4.7	6.1
Japan	31.2	3.4	-	7.0	1.4	1.7	2.9	6.6	45.8
Germany	8.6	0.8	2.5	-	15.8	28.7	4.7	21.5	17.4
France	9.4	0.7	1.4	30.2	-	17.3	5.6	22.8	12.6
UK	12.8	4.9	2.9	9.7	5.6	-	3.7	28.2	32.2
Italy	10.4	0.8	1.7	28.7	12.4	5.1	-	24.1	16.8
Other Europe	6.0	0.4	1.5	22.8	8.4	11.6	5.2	21.0	23.2
Imports, 1996									
United States	-	18.8	21.8	6.3	2.9	4.3	2.8	5.8	37.3
Canada	70.0	-	6.6	2.7	0.6	0.6	1.1	6.3	12.1
Japan	25.7	2.5	-	5.9	3.1	2.3	5.6	2.0	52.9
Germany	6.3	0.6	5.6	-	11.9	25.8	7.3	20.1	22.4
France	6.9	0.6	3.1	23.4	-	16.5	10.0	23.3	16.2
UK	10.7	1.1	5.6	18.0	8.3	-	6.0	30.6	19.7
Italy	5.0	0.6	2.3	25.8	12.8	6.5	-	26.4	20.6
Other Europe	8.1	0.5	3.0	19.5	10.4	10.5	6.8	22.9	18.4

[b] Interpretation : In 1970, 24.7% of the US import came from Canada. In Marmotte, this share corresponds to SM_{ca}^{us} (the share of *ca* in the imports of *us*).

d) Export equation

As for imports, the export equation is an error correction model. In the long run, exports are an increasing function of a foreign demand indicator and an indicator of competitiveness of a given country defined as the ratio between the price of the competitors on export markets and the price of the country's exports. The specification of the export equation is as follows:

$$
\begin{aligned}
(4)\qquad Log\left(EXM_{t}/EXM_{t-1}\right) &= exm0 + exm1.Log\left(FACT_{t}/FACT_{t-1}\right) \\
&+ exm2.Log\left((PFM_{t}\ /\ PXM_{t})/(PFM_{t-1}/PXM_{t-1})\right) \\
&+ exms.\begin{bmatrix} Log(EXM_{t-1}) - exm.Log(FACT_{t-1}) \\ - exm3.Log\left(PFM_{t-1}/PXM_{t-1}\right) \end{bmatrix}
\end{aligned}
$$

where *EXM* are exports of goods and services in constant price 1990, *FACT* is the world demand in constant price 1990, *PFM is* the price of competitors on export markets in national currency.

The foreign demand addressed to the country i is defined as the sum of imports of its trade partners weighted by the share of exports of country i in the total imports of each partner j (SM_i^j).

$$(5)\ FACT_i = \sum_{j \neq i}^{N} SM_i^j.IMM_j$$

In the export equations also enter competitors prices of the country on export markets. To define the competitor price of country i, we have applied the so-called « double-weighting » method used in particular by the OECD. This method takes account of the structure of competition in both export and import markets (see weighting matrices reported in Annex Table 1). A discussion of this methodology is given in Durand *et al.* (1992). For each year, starting in 1970, the procedure calculates for a given country the relative importance of its competitors in the domestic and foreign markets (which is determined by the pattern of supply on that markets), and then weights it according to the relative share of the different markets in the total demand directed at this country.

More precisely, in a first step, we compute the export price of the country i's competitors on the market j. Let's note k the competitor countries and $\left(PXM_i^j\right)$ this price, we write:

$$(6)\ LogPXM_i^j = \sum_{k \neq i}^{N} \left(\frac{SM_k^j}{1 - SM_i^j} \right) * Log\left[\left(\frac{E_i/E_k}{E_i^{90}/E_k^{90}} \right) * PXM_k \right]$$

where k includes the rest of the world and with E_i, the exchange rate of country i and E_i^{90} the exchange rate of i in 1990. This price corresponds to the sum of export prices of the countries k, weighted by the share of exports of each country k in total imports of country j corrected of exports coming from country i $\left(\frac{SM_k^j}{1 - SM_i^j} = \frac{X_{k \to j}}{M_j - X_{i \to j}} \right)$.

The country's i competitor price on the export markets as a whole $\left(PFM_i\right)$ accounts for the geographic structure of exports of country i, i.e. the share that the market j represents in the total exports of country i $\left(WM_i^j = \frac{X_{i \to j}}{X_i} \right)$. These shares are displayed for 1970 and 1996 in Table 1.

(7) $LogPFM_i = \sum_{j \neq i}^{N} WM_i^j.Log(PXM_k^j)$ where j includes the rest of the word.

e) export price equation

The export price equation is an error correction model, where, in the long run, export price is a geometric average of GDP deflator at factor costs and foreign competitor prices. This point is detailed in Appendix 1.

(8) $$\begin{aligned} Log(PXM/PXM_{-1}) &= pxm0 + pxm1.Log(p / p_{-1}) + pxm2.Log(PFM/PFM_{-1}) \\ &+ pxms.[Log(PXM_{-1}) - (1 - pxm3)Log(P_{-1}) - pxm3.Log(PFM_{-1})] \end{aligned}$$

The long-term relationship indicates whether the export price setting of a country depends more on the competitor price than its production price (*pxm3*>0.5).

The rest of the world export price for non-primary goods is simply indexed to the US GDP deflator:

(9) $\log(PXM_{RW}) = \log(P_{US}.E_{RW} / E_{RW}^{90})$

f) World trade discrepancies

At this stage no mechanism adjusts the world total of imports to the world total of exports. Hence such a mechanism has been introduced in an *ad hoc* but usual way. The surplus of total imports on total exports in volume (*WTMR*) is allocated between the exports of each modelled country in proportion to their share in world exports (*wtsm*).

The world trade discrepancy in non primary goods and services and in constant dollar is defined as:

(10) $$WTMR = \sum_i (IMM_i - EXM_i) / E_i^{90} + \left(IMM_{RW} - EXM_{RW} \frac{P_{US}}{P_{US}^{SS}} \right) / E_{RW}^{90}$$

with i=1 to 17 (17 industrialised countries), where *RW* denotes the rest of the world and P_{US}^{SS} is the level of the steady state US price.

The exports in volume of the non primary goods are adjusted for real world trade discrepancy:

(11) $$EXMA_i = EXM_i + \frac{wtsm_i}{(1 - wtsm_{RW})}.WTMR.E_i^{90}$$

Once this correction has been made, a world trade discrepancy in non primary goods and services, in current dollar (*WTM*) is left between the sum of imports and exports in value:

$$\begin{aligned} WTM_t &= \sum_i (PMM_i.IMM_i - PXM_i.EXMA_i) / E_i \\ &+ \left(IMM_{RW} PMM_{RW} - EXM_{RW} PXM_{RW} \frac{P_{US}}{P_{US}^{SS}} \right) / E_{RW}^{90} \end{aligned}$$

with i=1 to 17 (17 industrialised countries).

The discrepancy is then allocated between the import prices of each country in proportion to their share in world exports:

(13) $$PMMA_i = PMM_i - \frac{wtsm_i}{(1-wtsm_{RW})}.WTM.E_i \ / \ IMM_i$$

with i=1 to 17 (17 industrialised countries).

The same adjustment is made with net exports in primary commodities and net foreign assets. This point is discussed more precisely in Appendix 2. A similar adjustment is realised for the net exports of primary commodities. The share of each country in world export (*wtsm* for the manufactured goods and services and *wtsc* for the primary commodities) are displayed for 1970 and 1996 in Table 2.

World trade discrepancy for primary commodities, in constant dollar:

(14) $$WTCR = \sum_i NXC_i \ / \ E_i^{90}$$

with i=1 to 18 (17 industrialised countries and the rest of the world).

Export volume - primary goods, adjusted for real world trade discrepancy:

(15) $$NXCA_i = NXC_i - \frac{wtsc_i}{(1-wtsc_{RW})}.WTCR.E_i^{90}$$

Table 2: share in world merchandise trade (in percentage)

	Goods and services (*wtsm*)		Primary commodities (*wtsc*)	
	1970	1996	1970	1996
United States	13.9	13.7	10.0	11.7
Canada	4.8	3.6	4.4	4.3
Japan	5.3	7.3	7.0	7.0
Germany	11.3	9.9	6.0	5.3
France	6.1	5.8	4.0	3.8
UK	6.8	5.6	4.3	3.2
Italy	4.5	4.7	4.0	3.0
Rest of Europe	16.9	15.4	10.9	11.1
Rest of the world	30.4	34.0	49.4	50.6

g) The treatment of the rest of the world

Marmotte does not include a model of the rest of the world. Currently, the imports and exports prices of the rest of the world are measured in dollars and exogenous[37]. The volume of imports measured in constant dollars is fixed. The volume of exports has a positive elasticity in relation

[37] Thus, the prices of the rest of the world are a nominal anchor for the US, and American prices do not present an hysteresis. This reduces the number of unitary eigenvalues in the dynamic model and the dimension of the space of indetermination of the steady state model, by one.

to the ratio of the production price in the US and the production price in the rest of the world, which is exogenous. Thus, a price increase in the US improves the trade balance of the rest of the world, measured in current dollars or in constant dollars. These assumptions are far from satisfactory, and may creates unwanted features in simulation results. In the future we will introduce a model of the rest of the world, based on the choices made by Multimod Mark 3.

III.2 Results of estimations

Two equations have been estimated econometrically: the export equation (4) and the export price equation (8). The other equations are either calibrated or have simply the status of definition. Calibrated parameters for the imports equation has been introduced for reason of stability in Marmotte.[38] This part presents the results of the estimation realised on the panel for the 17 countries of Marmotte over the period 1970-1996. Data for exports and weighted share are from the database Chelem of CEPII. Prices and final demand are from the OECD Economic Outlook.

Since all the explanatory variables are assumed to be exogenous, the parameters can be estimated by the SUR method. First, we estimate the system by the non-linear least square by gathering all the observations. The residuals obtained make possible the computation of the covariance matrix which is used, once corrected for autocorrelation, as the initial matrix for the following iterations. Like the other empirical works related with Marmotte (see Final appendix 1), we introduce the procedure of factor analysis of the covariance matrix and the choice of the parameters is made from nested tests.[39]

a) Exports

The export equation was first estimated with a long-term demand elasticity which was not constraint to be equal to one. The estimations led to elasticities close to one and the tests had not rejected the constraint. As a consequence, the specification retained here includes this constraint.

The results have led to retain two models in which one parameter is constrained to be equal across countries: in the first one, the constrained parameter is the convergence speed (*exms*) is the same across countries whereas in the second one, it is the long-term price elasticity (*exm*). In order to choose between these two models, we have implemented a nested test strategy, based on a "J test" (Davidson and MacKinnon, 1993; Greene, 1997). The results have not been able to solve the problem. In these circumstances, economic arguments must help for the choice. Since the studied countries belong to the OECD, they are industrialised, open to both trade and financial flows. Hence, in the long run, due to the effect of competitiveness and catching up, it is reasonable to consider that these countries have the same behaviour. The model retained is the one with the same long-term price elasticity for all the countries. The differences across countries refer to the dynamics of the export equations. On one side, in the

[38] Estimated parameters for import equations highlight strong structural differences between countries both in the short and in the long run. These parameters introduced some stability problems in Marmotte; they have thus been discarded and replaced by calibrated parameters.

[39] The different steps to implement the nested tests can be found in Hervé (2001).

short run these countries have different reactions to shocks affecting their economy and on the other side, they do not converge at the same pace to the long run equilibrium.

The estimation results are displayed in Table 3. The short-term elasticities have all the expected sign. However, for Denmark, Ireland, and Japan, these elasticities are non-significant. The long-term relationship can only explain the exports of these countries. For France, Italy and Portugal, the short-term price elasticities are not significant. The short-term demand elasticities are less than one, implying that export are less sensitive to a change in demand in the short term than in the long term.

The convergence speed is non-significant for Austria whereas in the other cases, it is significant and with the right sign. Differences among the convergence speeds range from -0.09 for Ireland and–0.60 for Denmark. Finally, the long-term price elasticity is the same across countries and is relatively high (1.11).

For Ireland and Portugal, the R^2 is very low indicating that for these countries the retained model is not the best one. We can interpret this result as an indication of the gap between these countries and the model that is suitable for most of the countries of Marmotte. These results are not contradictory with the reality since these two countries have different specialisation, which are mainly based on direct investment and trade of re-exports.

The results show that for the countries of Marmotte, the differences are less pronounced for exports than for imports. In the long run the paths would be the same. However, the convergence speeds being different, these countries do not converge at the same pace to the long run relationship.

b) Export prices

The second estimated equation concerns the export prices. The best model[40] is the model for which all the parameters are the same across countries except the short-term elasticity to the GDP prices. Results are displayed in Table 4.

In the short term, the export price sensitiveness to competitors' price changes is the same for all the countries (0.39). Besides, the exporters are more sensitive in the short run. Hence, part of the response of the exporters to an increase in the production costs is related with margin behaviours (exporters prefer to pass the cost increase on to its prices even if it means competitiveness decrease). This is especially verified for the US, Japan, and Canada whose elasticities are respectively 0.94, 0.83 and 0.8. In Finland, Spain and Portugal, the impacts of production price changes are also larger on export prices than competitors' price changes. However, the production price elasticity is smaller than for the other countries.

In the long run, we observe the opposite phenomenon. The exporters set their prices on the competitors' in order to make their competitiveness stable and preserve their market shares, the export price elasticity to competitors' prices being equal to 0.68. This result is quite surprising especially for the US which are considered as a price maker. However, as regards to the R^2 statistics, we can see that for three countries (Greece, the US and Canada), this model is not as suitable as for the others.

[40] We have also estimated models with outputs gaps. However, the elasticities were very small and we have not retained this specification.

Table 3: Estimation results for the export equation

Countries	exm0	exm1	Exm2	exms	Exm	Exm3	R²
Austria	-0.006	0.80	0.37	-0.05	1	1.11	0.57
	ns	(5.33)	(2.64)	ns		(18.59)	
Belgium	-0.09	0.53	0.46	-0.21	1	1.11	0.40
	(-3.14)	(2.85)	(2.56)	(-4.38)		(18.59)	
Netherlands	-0.12	0.88	0.13	-0.26	1	1.11	0.77
	(-7.54)	(11.75)	(1.90)*	(-8.38)		(18.59)	
Denmark	-0.25	0.11	0.27	-0.60	1	1.11	0.23
	(-3.01)	ns	ns	(-4.00)		(18.59)	
Finland	-0.19	0.64	0.37	-0.28	1	1.11	0.47
	(-2.56)	(2.52)	(2.42)	(-2.69)		(18.59)	
Sweden	-0.25	0.93	0.28	-0.41	1	1.11	0.77
	(-6.11)	(7.69)	(3.17)	(-6.21)		(18.59)	
Italy	-0.29	1.63	-0.005	-0.34	1	1.11	0.34
	(-4.33)	(2.60)	ns	(-4.50)		(18.59)	
Spain	-0.09	0.72	0.65	-0.35	1	1.11	0.13
	(-2.94)	(3.94)	(2.97)	(-4.75)		(18.59)	
Portugal	-0.06	0.65	0.09	-0.22	1	1.11	0.05
	(-1.54)	(1.55)**	ns	(-3.83)		(18.59)	
Greece	-0.03	0.65	0.92	-0.22	1	1.11	0.32
	ns	(2.91)	(3.66)	(-2.40)		(18.59)	
Germany	-0.16	0.87	0.42	-0.23	1	1.11	0.72
	(-2.89)	(7.47)	(4.34)	(-2.99)		(18.59)	
France	-0.10	0.20	0.15	-0.27	1	1.11	0.48
	(-3.08)	(1.52)**	ns	(-4.41)		(18.59)	
United-	-0.09	0.57	0.20	-0.19	1	1.11	0.55
Kingdom	(-3.47)	(4.78)	(2.33)	(-4.47)		(18.59)	
Ireland	0.03	0.14	0.27	-0.09	1	1.11	0.05
	ns	ns	ns	(-2.67)		(18.59)	
Japan	-0.52	0.31	0.38	-0.60	1	1.11	0.43
	(-3.78)	ns	ns	(-4.11)		(18.59)	
US	-0.24	0.76	0.48	-0.52	1	1.11	0.78
	(-7.36)	(5.74)	(7.89)	(-8.71)		(18.59)	
Canada	-0.003	0.93	0.30	-0.16	1	1.11	0.74
	ns	(12.56)	(5.72)	(-4.39)		(18.59)	

III.3 Conclusion

The estimated elasticities of foreign trade equations have been largely discussed in the empirical literature. In particular, the estimation of price elasticities has been at the center of debates concerning the role of exchange rate in the change in trade balance in the long run. Our estimations have implied elasticities that are in line with other works. For the price elasticities, the results are satisfying since our estimates are larger than those found in other empirical works and hence more consistent with both the theory and the reality. Besides, the sum of the price elasticities is larger than one, verifying then the Marshall-Lerner conditions. However, some elasticities must be interpreted cautiously and should be discussed before their introduction in Marmotte.

In addition to these satisfying estimates, the econometric method has made possible to put forward several interesting results. The tests of the parameter equality have shown in particular that there is evidence of structural differences across countries. In the short-run, for both imports and exports, the sensitiveness differences across the studied countries are very important. In the long term, the differences are strong only for the import equation.

APPENDIX 1: The justification of the price equation of exports

In Marmotte we assume that firm i sells its good at price P_i. These prices aggregate to define a GDP deflator P. In a symmetric equilibrium all prices, P_i and P are equal. Residents who buy consumption goods or investment goods pay for them the absorption price PA. This price is a combination of the GDP deflator and the import price *PMMA*. The formula giving this combination of prices is deduced from a production function, which combines imports and national goods to get a final good, which can be either consumed or invested. The formula aggregating prices is the dual of this production function. The first order condition of the minimisation of the cost of the final good, gives the share of imports in total absorption. Actually, in Marmotte we do not fully respect the consistency, which should prevail between the equation giving the volume of imports and the equation giving the absorption price. However, we think that this inconsistency puts some flexibility in the model and is without serious consequences.

We could do the same with exports. The problem is that we want export price PXM to be a function of GDP deflator P, and of the price of foreign competitors PFM, not of the price of imports. A solution to get this result is to assume that French firms face two markets, the national one and the international one, and that their competitors differ between these markets. Modelling a discriminating monopoly behaviour of firms would be cumbersome, and we will suggest here a different solution. Its advantage is that it is consistent and it concludes to a specification, which is quite, agrees with the one used in macro econometric models.

We assume that total exports are a function of the ratio between their price and the price of foreign competitors:

(A1.1) $EXM = B(PXM/PFM)^{-b}$, with: $b > 1$.

Exports are made by n exports firms, indexed by i, which compete *à la Cournot*. In order to export EXM_i firm i must use a variable intermediary consumption of national good Q_i, with:

(A1.2) $EXM_i = AQ_i^a$, with: $0 < a < 1$.

Moreover the firm must use a fixed intermediary consumption of national good c. The firm determines the volume of its exports X_i by maximising its profit:

(A1.3) $\Pi_i = \left[(EXM_i + EXM_{-i})/B\right]^{-1/b} PFM * EXM_i - P(EXM_i/A)^{1/a} - cP$,

where EXM_{-i} represents the exports of other firms, which are assumed to be exogenous to the choice of firm i. The first order condition of the program is:

(A1.4) $-(PXM / b)EXM_i / EXM + PXM - (P / aA)(EXM_i / A)^{1/a-1} = 0$.

In the symmetric equilibrium, the exports of each firm are equal and: $EXM_i = EXM / n$. Then we get:

(A1.5) $PXM / P = [bn / (bn - 1)](1 / aA)(EXM / nA)^{1/a-1}$.

If we substitute in this equation the expression of EXM given by the export function (A1.1), we get the expression of the price of exports:

(A1.6) $PXM = \{[bn^2 / (bn-1)](B / An)^{1/a} / aB\}^{a/[a+b(1-a)]} P^{a/[a+b(1-a)]} PFM^{b(1-a)/[a+b(1-a)]}$ The price of exports depends on the number of export firms. This number is endogenous. If we substitute equation (A1.5) in the definition of the profit of firm i (A1.3) we get:

(A1.7) $\Pi_i = PA^{-1/a} (EXM / n)^{1/a} [(b / a)n / (bn - 1) - 1] - cP$.

Now we substitute the exports demand (A1.1) in (A1.7):

(A1.8) $\Pi_i = PA^{-1/a} n^{-1/a} B^{1/a} (PXM / PFM)^{-b/a} [(b / a)n / (bn-1) - 1] - cP$.

Finally we substitute in this equation, equation (A1.6):

(A1.9) $\Pi_i = P(B / An)^{1-b} [(b / a)n / (bn - 1) - 1]\{(P / PFM)(b / a)n^2 / [B(bn-1)] / B\}^{-b/[a+b(1-a)]} - cP$

Then, we assume free entry, which implies that this profit must be equal to zero. This condition determines the number of exports firms n *(Existence and uniqueness not proved)*. n cannot be explicitly computed, but it depends on P / PFM. We substitute its value in the export price equation and we get:

(A1.10) $PXM / PFM = f(P / PFM)$, where f is an increasing concave function (These properties not proved). In Marmotte we approximate this function by a power function:

(A1.11) $f(P / PFM) = D(P / PFM)^g$, with: $0 < g < 1$.

The added value of the exports sector is zero. So we do not have to take it into account in the goods market equilibrium.

APPENDIX 2: The justification of the treatment of world trade discrepancies

We consider a world of n countries indexed by i or j. Each country produces a specific commodity. The equilibrium of the market of commodity i can be written:

(A2.1) $q_i = d_i + \sum_{j \neq i} m_{ij}$.

q_i is the output of country i. d_i is the demand for commodity i by country i, and m_{ij} represents the imports of commodity i by country j. We can deduce from this equation the equilibrium condition in value of country i:

(A2.2) $p_i q_i + \sum_{j \neq i} p_j m_{ji} = p_i d_i + \sum_{j \neq i} p_j m_{ji} + p_i \sum_{j \neq i} m_{ij}$.

p_i represents the production price of commodity i. We introduce the following definition equations:

(A2.3) $pa_i D_i = p_i d_i + \sum_{j \neq i} p_j m_{ji}$,

(A2.4) $D_i = d_i + \sum_{j \neq i} m_{ji}$,

(A2.5) $pm_i M_i = \sum_{j \neq i} p_j m_{ji}$,

(A2.6) $M_i = \sum_{j \neq i} m_{ji}$,

(A2.7) $X_i = \sum_{j \neq i} m_{ij}$.

D_i represents the absorption of country i, M_i are its imports and X_i its exports, all three measured in volume. pa_i is the absorption price, and pm_i the price of imports. The equilibrium condition in value (A2.3) can be rewritten:

(A2.8) $p_i q_i + pm_i M_i = pa_i D_i + p_i X_i$.

This equation is the same as equation (1) of the model. Equation (A2.5), which can be rewritten as: $pm_i = \sum_{j \neq i} (m_{ji} / M_i) p_j$, is equivalent to equation (22) of the model. However, in this last equation the ratios m_{ji} / M_i are fixed. So, equation (22) is only an approximation.

If we use equations (A2.2), (A2.4) and (A2.5) we can rewrite equation (A2.3) as:

(A2.9) $pa_i = (p_i q_i + pm_i M_i - p_i X_i)/(d_i + M_i)$.

As equation (A2.1) implies that: $d_i = q_i - X_i$, we can rewrite equation (A2.9) as:

(A2.10) $pa_i = (p_i q_i + pm_i M_i - p_i X_i)/(q_i + M_i - X_i)$.

Thus, we get the equilibrium condition in volume:

(A2.11) $D_i = q_i + M_i - X_i$.

This last equation is the same as equation (4) of the model. Now, equations (A2.5), (A2.6) and (A4.7) imply that both identities must be verified:

(A2.12) $\sum_i pm_i M_i \equiv \sum_i p_i X_i$,

(A2.13) $\sum_i M_i \equiv \sum_i X_i$.

Equation (A2.13) implies that the $2n$ imports and exports functions are not independent. We take this fact into account by choosing first these functions independent, then by allocating the disequilibrium of equation (A2.13) so as to correct the $2n$ initial functions.

If equation (A2.5) giving the prices of imports was satisfied, identity (A2.13) would imply identity (A2.12). But as equation (A2.5) is substituted by an approximation, disequilibrium arises in identity (A2.12). We remove it by changing the price of imports in comparison with the values given by the formula of the model.

IV INTEREST RATES ANS EXCHANGE RATES

In Marmotte, there are three interest rates per country: the short term, the long term interest rate, linked to the short term one by a term structure equation, and the firms' discount rate. The firms' discount rate is equal to the short term interest rate augmented by a risk premium, which is exogenous. The short term interest rate is defined by a monetary rule. The central bank goal is to stabilise inflation rate around target according to a Taylor monetary rule: the nominal interest rate increases with the inflation rate.

(1) $$(1+i_t) = (1+r^*)(1+p^*) + a\left(\frac{p_t}{p_{t-1}} - 1 - p^*)\right)$$

where r^* and p^* denote respectively the steady state value of the real interest rate and inflation rate. p_t is the production price.[41] The parameter α is calibrated as in Taylor (1983) and is equal to 1.5; hence if the inflation rate increases by one point, the real interest rate increases

[41] The production price is introduced in the monetary rule, rather than the consumer price, because the production price is devoted to provide a nominal anchor to the other prices, which depend to the production price, like absorption price, consumer price, GDP deflator…

by 0.5 point. The inflation rate entering the Taylor rule is the current inflation and not the expected inflation.

Box 1: why using a backward monetary rule?

Using a forward monetary rule would entail a nominal indeterminacy not only of the steady state value of reduced prices but also for the pattern of prices.[42] The backward characteristics of the rule is crucial to ensure a nominal anchor. Let P_t be the reduced price, that is the price deflated by its steady state inflationary trend; we then have the following relationship:

(B1) $$\frac{p_t}{p_{t-1}} = \frac{P_t}{P_{t-1}}(1+\boldsymbol{p}^*)\ .$$

The steady state level for the nominal interest rate is defined by the identity: $(1+\bar{i}) = (1+r^*)\dfrac{p_{t+1}}{p_t}$. Introducing the reduced price in the Taylor rule at the steady state yields:

(B2) $$(1+r^*)\left(\frac{P_{t+1}}{P_t}-1\right) = \boldsymbol{a}\left(\frac{P_t}{P_{t-1}}-1\right)$$

On the assumption that $\boldsymbol{a} > 1+r^*$, the level of the reduced price is then given by the backward equation:

(B3) $$P_t = P_{t-1}$$

Hence, the backward rule introduces an hysteresis in the model. The nominal anchors which determine the output deflators in the different countries are prices inherited from the past. The steady state model has thus an infinity of solutions for prices, exchange rates and nominal variables in level, which belong to a liner variety with a dimension equal to the number of independent central banks. Although the prices level at the steady state is undetermined, the dynamic pattern is well determined by the level of prices inherited from the previous period.

This very simple rule of monetary policy is assumed for all the countries except for the United States. For the countries belonging to the EMU, it is assumed that the European central bank controls for a geometric average of inflation rates, weighted by the share of GDP of each country to the total GDP of the euro area, denoted by *w*.

(2) $$(1+i_t) = (1+r^*)(1+\boldsymbol{p}^*) + \boldsymbol{a}\left(\prod_{i=1}^{N}(\boldsymbol{p}_t^i - 1)^w - \boldsymbol{p}^*\right)$$

[42] See Beffy and Laffargue for a formal proof.

The currencies of the countries outside the euro area are perfectly flexible vis-à-vis the US dollar, which is the reference currency of the model. Between the EMU countries, exchange rates are fixed and a single exchange rate is defined: the euro/dollar. The exchange rates are determined by the uncovered parities, which include a country-specific risk premium.

(3) $$(1+i_t) - RP_t = (1+i_t^{US})\frac{e_{t+1}}{e_t} - 1$$

where e_t is the number of domestic currency unit per US dollar and RP is the risk premium. The assumption of perfect foresight in Marmotte entails that the expected exchange rate at time t is the exchange rate observed at time t+1.

Each country may have a different real interest rate, reflecting different degree of impatience in consuming today versus tomorrow. To avoid violating the stability of exchange rates in the steady state, we must amend the uncovered interest rate parity by adding a risk premium that will "punish" (or "reward") a country for its greater (or smaller) impatience. This premium is related to the external asset position of the country.

The risk premium is modelled as follows:

(4) $$RP_t = -\upsilon F_{t+1} e_t / (py_t Y_t)$$

where F_{t+1} is the net foreign asset position hold by the country at the beginning of period t+1, py is the GDP deflator at market price and Y is the GDP.

If the external indebtedness of a country increases, a higher risk premium must be attached to the currency of that country. In the dynamic model, as exchange rate affects the import and export equations, any increase of external indebtedness (reflecting a greater impatience from the consumers) will have a depressing effect on imports and the opposite effect on exports.

In the case of US, since the dollar is used as reference currency in the interest parities, the premium is modelled to affect US interest rates via the Taylor rule in US:

(5) $$(1+i_t^{US}) = (1+r^*)(1+\pi^*) + \alpha\left(\frac{p_t}{p_{t-1}} - 1 - \pi^*\right) - \frac{\upsilon F_{t+1}}{py_t . Y_t}$$

So, it is assumed that in addition to inflation the Fed also cares about the US net foreign asset position. In the dynamic model, as interest rate enters the consumption function, any increase of external indebtedness (reflecting a greater impatience from the consumers in US) will have a depressing effect on consumption.

CHAPTER IV

SIMULATION METHODOLOGY AND EIGENVALUES

I A SUFFICIENT CONDITION FOR THE EXISTENCE AND THE UNIQUENESS OF A SOLUTION PATH IN A MACRO-ECONOMETRIC MODEL[43]

As many recent macro-econometric models, for instance the multinational model of the IMF, Multimod Mark 3, or the model of the European Commission, Quest 2, Marmotte assumes perfect foresight. This choice was made possible by the development of simulation algorithms which are at the same time powerful and easy to use. For instance, an efficient relaxation algorithm was implemented by Juillard (1996) in the software Dynare which works under Gauss, and by Juillard and Hollinger in the command Stacks of Troll. However, the existence and the uniqueness of a solution for such models are not *a priori* warranted

Blanchard and Kahn (1980) established conditions for the existence and uniqueness of a solution, which are especially easy to check in terms of eigenvalues computed at the steady state of the model. However, these conditions only apply to linear models, the coefficients of which do not depend on time, and such that the exogenous variables can be assumed to be constant after some time.

Thus, this framework is very far from the features of large macro-econometric models. To be able to use the results by Blanchard and Kahn, it is first required that the model determines a balanced growth path. It is not necessary that this path represents a realistic approximation of history, and it is known that industrialised and developing economies do not grow in the neighbourhood of a balanced growth path[44]. We just want, for the model to be economically consistent, that it can generate after an horizon which may be in a distant future, a reasonable balanced growth path[45].

When this condition is satisfied, it is possible to compute the linear approximation of the model around its balanced growth path, and the solution of the model may be required to converge to this path when time increases indefinitely. This condition will be called stability in the absolute difference. However, some of the coefficients of the linear approximation appear as geometric functions of time, and the results by Blanchard and Kahn can not be applied. However, they can be applied if all the variables are put on a common trend. If this trend has a zero growth rate, the model is said to have been written in reduced variables, and its stability is required in the relative difference. If the growth rate of the common trend is the highest balanced growth rate present in the model, the linear approximation of the model is said to have been written in

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[43] The first part of this Chapter is based on Laffargue (2000).

[44] For example, Maddison (1996) shows that for all industrialised countries over the period 1973-1991, productive capital increased at a faster rate than production. Foreign trade also increased faster than output, and the various sectors of the economy exhibited very contrasted trends.

[45] The horizon after which we can reasonably assume that the economy follows a balanced growth path is high, let us say 20 years for a model poor in demographic variables, much more for a model richer in this respect.

expanded variables, and its stability is required in the expanded difference. In both cases, the results by Blanchard and Kahn can be applied. If they are satisfied for the model written in reduced variables and for its linear approximation written in expanded variables, then the model determines a unique solution stable in the absolute difference, at least in the neighbourhood of its balanced growth path.

The first paragraph presents the results by Blanchard and Kahn, with its extension to the case of hysteresis, which was developed by Giavazzi and Wyplosz (1986). The second paragraph establishes the sufficient local conditions of existence and uniqueness that we propose.

I.1 The Blanchard et Kahn's conditions: A reminder[46]

A dynamic linear model with perfect foresight and with coefficients independent of time can always be written in the following form:

$C_1 y^1_{t-1} + C_0 y_t + C_{-1} y^2_{t+1} = U_t$, $t \geq 1$, with C_0 non-singular.

To get this form we may have to introduce artificial variables to eliminate variables appearing with a lag or a lead greater than one, and to prevent a given variable from appearing simultaneously with leads and with lags. The endogenous variables belong to one of the three mutually exclusive classes which follow: n_1 variables appear in a contemporary or lagged form; They are denoted as predetermined. n_2 other variables appear in a contemporary or lead form; They are denoted as anticipated. The n_3 last variables only appear in a contemporary form; They are denoted as static. These three categories of variables constitute at time t the column vectors y^1_t, y^2_t and y^3_t. The piling up of these vectors in the order: y^2_t, y^3_t and y^1_t defines the vector of the endogenous variables y_t of dimension: $n = n_1 + n_2 + n_3$.

Let us denote by: C^2_0, C^3_0 and C^1_0 the three matrices with respectively n_2, n_3 and n_1 columns. The concatenation of these matrices gives the matrix $C_0 : C_0 = (C^2_0 \vert\; C^3_0 \;\vert C^1_0)$. Let us make the change of variables: $x^1_t = y^1_t$, $x^2_t = y^2_{t+1}$, $x^3_t = y^3_t$, and let us denote the piling up of these vectors in the order: x^3_t, x^1_t, x^2_t, by x_t. Then, the model can be rewritten:

$$C_1 x^1_{t-1} + C^2_0 x^2_{t-1} + (C^3_0 \vert\; C^1_0 \;\vert C_{-1}) x_t = U_t .$$

In general, for x^1_{t-1} and x^2_{t-1} given, equation (2) does not determine a unique value for x_t: To get this absence of uniqueness it is sufficient that some anticipated variables appear in a led form always in the same linear combination. However, it is possible to make a series of eliminations and transformations of anticipated variables to put the model in the case where the uniqueness of x_t is warranted[47]. Thus, it is not restrictive to consider, in the rest of the section, the system, which could *a priori* look more specific:

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[46] We follow in this section the presentation of our paper of 1990.

[47] Sims (1997) and Juillard (1999) give a solution to this problem. It rests upon the computation of generalized eigenvalues based on a generalized decomposition of Schur. Then, the variables

(3) $\quad x_t = Ax_{t-1} + h_t,\ t \geq 1.$

We assume that all the static variables were eliminated. Then, the dimension of vectors x_t and h_t is $n_1 + n_2$, and A is a square matrix with the same dimension. The difficulty with system (3) is that, if it is justified to assume that the initial value of the predetermined endogenous variables: $x_0^1 = y_0^1$ is given, we cannot make the same assumption for the initial values of the anticipated endogenous variables: $x_0^2 = y_1^2$. However, it seems justified to require that if h_t is permanently fixed at a constant value $\overline{h}$, then there exists a unique path for the endogenous variables which tends to a finite value, which is the steady state of system (3), let be: $\overline{x} = (I - A)^{-1}\overline{h}$. We will see that this condition, which we will call stability of the model, implicitly defines x_0^2.

We will assume now that matrix A can be reduced to a diagonal form[48]. We will denote by Λ_1 the diagonal matrix of the $n_1^{'}$ eigenvalues of absolute values less than or equal to 1, and by W_1 the matrix of dimension $n_1^{'}(n_1 + n_2)$ of the associated left eigen-vectors[49]. We can deduce from (3):

(4) $\quad W_1 x_t = \Lambda_1^{'} W_1 x_0 + \sum_{j=1}^{t} \Lambda_1^{t-j} W_1 \overline{h}.$

We make assumption H_1 that $\overline{h}$ is not orthogonal to any row of W_1. Then, for $W_1 x_t$ to be bounded, there must not exist any eigenvalue of absolute value equal to 1 in Λ_1. We define Λ_2 and W_2 in a similar way for the eigenvalues with absolute values larger than 1, of number $n_2^{'}$. We make assumption H_2 that W_{22}, which is the matrix built with the $n_2^{'}$ last columns of W_2, is non-singular. We deduce from (3):

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with a lead which can be eliminated are as many as there are infinite eigenvalues. Juillard used this method in its Gauss software, Dynare (Juillard, 1996), and with Hollinger in the command Lkroot of Troll. We have used Dynare and Lkroot in our applications of the methodology developed in this paper.

[48] This means that if we have a multiple eigenvalue, we can associate to it the same number of eigen-vectors as its order of multiplicity. This assumption can be removed at the cost of a heavier presentation, which we have preferred avoiding (see the paper by Blanchard and Kahn).

[49] An eigenvalue of multiple order appears in Λ_1 as many times as its order of multiplicity, and a basis of the space generated by its eigen vectors appears in W_1. The same rule will be used later with Λ_2 and W_2.

(5) $$W_2 x_t = \Lambda_2^t (W_2 x_0 + \sum_{j=1}^{t} \Lambda_2^{-j} W_2 \bar{h}) .$$

For $W_2 x_t$ to be bounded, the expression in brackets must tend to zero when t increases indefinitely, let be:

(6) $$W_2 x_0 = -\sum_{j=1}^{\infty} \Lambda_2^{-j} W_2 \bar{h}$$

This condition is also sufficient, because in this case $W_2 x_t$ can be written:

(7) $$W_2 x_t = -\sum_{i=1}^{\infty} \Lambda_2^{-i} W_2 \bar{h} .$$

The value of x_0^1 and relation (6) constraint the initial state of the economy x_0. For a value of this vector verifying these restrictions, equations (4) and (5) determine a unique path converging toward the steady state of model (3) (the square matrix got by piling up W_1 and W_2 is regular and it can be easily checked that (3) is satisfied). For x_0 to exist and be unique, it is necessary and sufficient that: $n_2 = n_2^{'}$.

Proposition 1 (Blanchard et Kahn (1980)). Under assumptions H_1 and H_2, the necessary and sufficient condition for equation (3) to determine a unique and stable path, is that matrix A has as many eigenvalues of absolute values less than 1 as there exist predetermined endogenous variables, and as many eigenvalues of absolute values larger than 1 as there are anticipated variables.

We can make a more general assumption than H_1, which is that there can exist eigenvalues of absolute value equal to 1, if $\bar{h}$ is orthogonal to the eigen vectors related to these eigenvalues[50]. In general, this property results from exclusion relations in the structural form of the model. So, we can require that it is satisfied for all the vectors $\bar{h}$ having an economic meaning. Then, Proposition 1 stays valid, if we consider that n_1 represents the number of eigenvalues of absolute values less than *or equal to* 1. Let us consider in Λ_1 and W_1 a higher part Λ_1^* and W_1^* and a lower part Λ_1^{**} and W_1^{**}, respectively related to the eigenvalues of absolute values equal or less than 1. Their respective dimensions are n_1^* and n_1^{**}. Then, $W_1^{**} x_t$ and $W_2 x_t$ still tend to $W_1^{**} \bar{x}$ and $W_2 \bar{x}$ when t increases indefinitely. However, this property does not hold for: $W_1^* x_t = \Lambda_1^{*t} W_1^* x_0$. This permanent dependency of the path relatively to its initial

[50] This means that if an eigenvalue of absolute value equal to 1 is complex, $\bar{h}$ must be orthogonal to the real and imaginary parts of its associated eigen vectors.

conditions is called hysteresis, and characterises some economic mechanisms[51]. When there does not exist any eigenvalue with absolute value equal to 1, $\bar{x}$ is unique. Otherwise, it belongs to a linear variety with dimension n_1^*. More precisely, we can notice that, when the eigenvalues with an absolute value equal to 1 are real, x_t will tend to a point in this variety. When some of these eigenvalues are complex x_t will tend to follow a periodic path inside this variety. We have the proposition:

Proposition 2. Under assumption $H_1^{'}$, that $\bar{h}$ is not orthogonal to any of the rows of W_1^{**} and is orthogonal to W_1^*, and assumption H_2, the necessary and sufficient condition for equation (3) to define a unique and stable path, is that matrix A has as many eigenvalues of absolute values less than or equal to 1 as there exists predetermined endogenous variables, and as many eigenvalues of absolute values larger than 1 as there exists anticipated variables.

This problem of hysteresis was investigated by Giavazzi and Wyploz (1985). Most cases of hysteresis are associated with an eigenvalue equal to 1. First, we have the endogenous growth models where the level of production depends on its initial value forever. A different case of hysteresis is met in models of a closed economy where the inflation rate is determined in the long run by the monetary rule followed by the Central Bank. Quite frequently, this is the case for the rules put by Taylor and Furher and Moore, it links a nominal short-term interest rate to inflation rates and to indicators of the level of activity. Thus, the steady state of the model determines the inflation rate, but leaves undetermined the price level[52]. In the simulation of the dynamic model this last variable will remain dependent on its initial value forever. A model of an open economy will have the same property if the exchange rate is flexible, which will prevent the price level from being determined in the steady state by exogenous foreign prices. A multinational model where the rules of monetary policy would be relative to rates and not to levels, would have the same property of hysteresis: The number of price levels and consequently of exchange rates minus 1, undetermined in the steady state, would be equal to the number of monetary policy rules expressed in rates. Béraud (1998) gives another example of hysteresis associated with an eigenvalue equal to 1. She considers a neo-classical economy of two countries. Each country has specific utility and production functions. Both of them have the same discount rate and the international capital market is perfect. A first block of equations of the model determines the path followed by all the world aggregated variables and the allocation of production between the two countries. Hysteresis appears in a second block of equations which determines the distribution of the ownership of the world capital between the two countries, and consequently, the distribution of consumption. These distributions will permanently depend on the initial state of the economy. In all these examples, the eigenvalue equal to 1 appears when the model has been rewritten in reduced variables as we will notice in

[51] We can notice that in the case of hysteresis, stability does not refer to the endogenous variables of the model, but to the product of the eigen vectors non related to the unitary eigenvalue by the vector of the endogenous variables.

[52] Giavazzi and Wyplosz (1985) give an example of this kind of hysteresis. The result that the price level is undetermined when monetary policy sets the nominal interest rate, is due to Wicksell at the beginning of the century. Juillard (1999) investigates in a systematic way this kind of hysteresis for several monetary policy rules.

the following section. Boucekkine, Germain and Licandro (1997) consider a model where the eigenvalues with an absolute value equal to 1 are complex. Their model considers a putty-clay production technology embodied in a neo-classical growth model. When the utility of the consumers is linear relatively to their consumption, the replacement echo effect reproduces permanently the initial allocation of investment over the periods which precede the simulation. There is a series of complex eigenvalues with magnitude 1; their periods are equal to the duration of the replacement echo effect and to its harmonics. These eigenvalues represent a Fourier decomposition of the echo effect.

The previous results cannot be directly applied to the case of macroeconometric models with perfect foresight. Actually, these models present the following properties: a) There does not exist a steady state for the endogenous and the exogenous variables, but a balanced growth path; b) On this path the various variables do not have the same growth rate; c) The linear approximation of the model in the neighbourhood of a balanced growth path has coefficients which change over time; d) Hysteresis does not manifest by the identical reproduction of initial conditions over time for some variables or combinations of variables, but by an expansion at a geometric rate endogenous to the model.

I.2 Theoretical analysis

A perfect foresight model can be written[53]:

(8) $$F(y_t, y_{t+1}, y_{t-1}, x_t) = 0,\ y_0 \text{ given.}$$

F is a vector of n equations, y is the column vector of the n endogenous variables, x is the column vector of the m exogenous variables (which can include time). At time t, the model determines the current values of the endogenous variables y_t in function of the values of these variables which are anticipated for the future or which were observed in the past, y_{t+1} and y_{t-1}, and of the values of the exogenous variables x_t.

We will assume that this model can determine a balanced growth path. To get this property we assume that there exists two diagonal matrices g and h, with respective dimensions n and m, such that for all vector $\bar{x}$ belonging to a subset Ω of R^m, there exists (at least) a vector $\bar{y}$ of R^n, satisfying:

(9) $$F(g^t\bar{y}, g^{t+1}\bar{y}, g^{t-1}\bar{y}, h^t\bar{x}) = 0, \text{ for all } t \geq 0.$$

The diagonals of g and h are the growth rates plus 1 of, respectively, the endogenous variables and the exogenous variables. These diagonal terms are assumed to be equal or larger

[53] The result that we can write under this form a very general rational expectations model, when the random shocks are small enough so we can approximate the expected value of a function by the function of the expected values of its variables, was proven by Broze, Gouriéroux and Safarz (1989) and Laffargue (1990).

than 1. The initial values of the balanced growth paths of the endogenous and exogenous variables are related by:

(10) $$F(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x}) = 0 .$$

$(\bar{x}, \bar{y})$ will be called a steady state of the model[54] and (x_t^s, y_t^s) = $(h^t\bar{x}, g^t\bar{y})$ will be a balanced growth path. We can see that the existence of a balanced growth path requires that equation (10) has a solution for all $\bar{x} \in \Omega$. We will assume that this solution $\bar{y}$ belongs to a subset Φ of R^n. But it is also necessary for function F to exhibit an homogeneity property implying that relation (9) is satisfied when equation (10) is verified. This property can be written:

(11) $$F(g^t\bar{y}, g^{t+1}\bar{y}, g^{t-1}\bar{y}, h^t\bar{x}) \equiv F(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x})$$

for all $\bar{x} \in \Omega$ and $\bar{y}$ solution of (10) and all $t \geq 0$.

Let us denote by $F_1^{'}$, $F_2^{'}$ and $F_3^{'}$ the matrices of the partial derivatives of F relatively to the vectors of the endogenous variables appearing respectively without lead and lag, with a lead and with a lag. More precisely, $F_1^{'}$ represents a square matrix with dimension n, the rows of which refer to equations and the columns to the contemporaneous endogenous variables relatively to which the derivation was computed. $F_2^{'}$ and $F_3^{'}$ are defined in the same way, but for the variables appearing respectively with a lead or with a lag. Let us now compute a linear approximation of model (8) in the neighbourhood of a balanced growth path:

(12) $$\begin{aligned} &F_1^{'}(y_t^s, y_{t+1}^s, y_{t-1}^s, x_t^s)(y_t - y_t^s) + F_2^{'}(y_t^s, y_{t+1}^s, y_{t-1}^s, x_t^s)(y_{t+1} - y_{t+1}^s) \\ &F_3^{'}(y_t^s, y_{t+1}^s, y_{t-1}^s, x_t^s)(y_{t-1} - y_{t-1}^s) = 0 \end{aligned}$$

We deduce from this expression a first definition: The stability in the absolute difference is the tendency of every endogenous variable of the model to converge to its balanced growth path value when time increases indefinitely:

(13) $$y_t - y_t^s \rightarrow 0, \text{ when } t \rightarrow \infty$$

This definition gives the justification of the most natural way to simulate the model. Model (8) represents a system of finite difference equations with initial conditions (on the predetermined variables) and with final conditions (on the anticipated variables). We select a time horizon long enough and we choose as terminal conditions at this horizon the equality of the anticipated variables to their balanced growth values. Then, we can use the usual algorithms developed to solve this kind of mathematical problem.

However, in linear approximation (12), the matrices of the coefficients depend on time, so we cannot use the results by Blanchard and Kahn[55]. To be able to use these results we must first

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[54] In the case of hysteresis there exist several solutions. We can select any of them.

[55] Malgrange (1981) had already noticed this difficulty. More precisely, he proved that, under the homogeneity condition (14) which we will introduce later, the dynamic system (12) has eigen vectors which change and eigenvalues which remain constant over time. He shows on an example that eigenvalues alone can give very mistaken indications for stability. For instance,

strengthen the homogeneity property (11), in a way which does not look restrictive from a practical point of view. Then, we impose the following condition:

(14) $$F(g^t y_1, g^{t+1} y_2, g^{t-1} y_3, h^t x_1) \equiv k^t F(y_1, g\ \ y_2, g^{-1} y_3, x_1),$$

$\forall t \geq 0$, $\forall y_1, y_2, y_3 \in \Phi$ and $\forall x_1 \in \Omega$,

where k represents a diagonal matrix of dimension n.

This property of homogeneity has interesting implications. Let us differentiate identity (14) relatively to y_1, y_2 and y_3. We get identities:

(15) $$F_1^{'}(g^t y_1, g^{t+1} y_2, g^{t-1} y_3, h^t x_1) g^t \equiv k^t F_1^{'}(y_1, g\ \ y_2, g^{-1} y_3, x_1)$$

(16) $$F_2^{'}(g^t y_1, g^{t+1} y_2, g^{t-1} y_3, h^t x_1) g^t \equiv k^t F_2^{'}(y_1, g\ \ y_2, g^{-1} y_3, x_1)$$

(17) $$F_3^{'}(g^t y_1, g^{t+1} y_2, g^{t-1} y_3, h^t x_1) g^t \equiv k^t F_3^{'}(y_1, g\ \ y_2, g^{-1} y_3, x_1)$$

Then, the linear approximation (12) can be rewritten, after simplifying by k^t:

(18) $$\begin{aligned} & F_1^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x}) g^{-t}(y_t - y_t^s) + F_2^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, x) g^{-t}(y_{t+1} - y_{t+1}^s) \\ & F_3^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x}) g^{-t}(y_{t-1} - y_{t-1}^s) = 0 \end{aligned}$$

We can deduce from this expression two new definitions of stability. For the first, called in the relative difference, we define the vector of the reduced endogenous variables by: $y_t^{'} = g^{-t} y_t$. Then, equation (18) can be rewritten in reduced variables:

(19) $$\begin{aligned} & F_1^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x})(y_t^{'} - \bar{y}) + F_2^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x}) g (y_{t+1}^{'} - \bar{y}) \\ & F_3^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x}) g^{-1}(y_{t-1}^{'} - \bar{y}) = 0 \end{aligned}$$

Stability is defined as the convergence of the vector of the reduced endogenous variables to the steady state:

(20) $$y_t^{'} - \bar{y} = g^{-t}(y_t - y_t^s) \rightarrow 0, \text{ when } t \rightarrow \infty$$

The matrix of the coefficients does not depend on time any more, so we can apply the results by Blanchard and Kahn. We must notice that condition (20) is less strict than condition (13). Thus, it is more likely, under this condition, to get a solution of the model. But it is also more likely to get an infinity of solutions.

---

when the model only includes current and lagged variables, we can have all the eigen values with magnitudes smaller than 1 and a divergent path of the economy. Malgrange also proved that the requirement of the tendency to zero of the relative differences of the variables to their values of balanced growth, is equivalent to the stability in the relative difference which will be defined later.

A remark, which has interesting practical consequences, is that relation (19), which is the writing in reduced variables of the linear approximation of the model computed with its original variables, can also be obtained as the linear approximation of the model directly written in reduced variables. To show that let us define the vector of the reduced exogenous variables by: $x_t^{'} = h^{-t} x_t$. Then, model (8) can be rewritten:

(21) $$F(g^t y_t^{'}, g^{t+1} y_{t+1}^{'}, g^{t-1} y_{t-1}^{'}, h^t x_t^{'}) = 0$$

and, if we use the homogeneity condition (14):

(22) $$F(y_t^{'}, g y_{t+1}^{'}, g^{-1} y_{t-1}^{'}, x_t^{'}) = 0$$

The linear approximation of this equation is identical to equation (19).

For a second definition of stability, called in the expanded difference, we must first introduce the diagonal matrix of dimension n, with as generic element the highest balanced growth rate among those appearing in matrix g. We call this matrix $g_{\max}$. We multiply equation (18) by $g_{\max}^t$, and we define the vector of expanded endogenous variables by: $y_t^{''} = g_{\max}^t g^{-t} y_t = g_{\max}^t y_t^{'}$. Stability requires that these expanded endogenous variables converge to their balanced growth path $g_{\max}^t \bar{y}$. Thus:

(23) $$y_t^{''} - g_{\max}^t \bar{y} = g_{\max}^t g^{-t} (y_t - y_t^s) \to 0 \text{, when } t \to \infty$$

We see that this condition is stricter than condition (13). Thus it is less likely, under this condition, to find a solution of the model. But it is also less likely to find an infinity of solutions. The linear approximation of the model can be written:

(24) $$\begin{aligned} &F_1^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x})(y_t^{''} - g_{\max}^t \bar{y}) + F_2^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x}) g_{\max}^{-1} g (y_{t+1}^{''} - g_{\max}^{t+1} \bar{y}) \\ &F_3^{'}(\bar{y}, g\bar{y}, g^{-1}\bar{y}, \bar{x}) g_{\max} g^{-1} (y_{t-1}^{''} - g_{\max}^{t-1} \bar{y}) = 0 \end{aligned}$$

This time yet, the matrix of the coefficients does not depend on time, and it is possible to use Blanchard and Kahn's results. It is interesting to notice that equation (23) cannot be considered as the linear approximation of the model written in expanded variables, except if we strengthen the homogeneity condition (14).

We show in the Appendix that the eigenvalues of the linear approximation of the model written in expanded variables are equal to $g_{\max}$ times the eigenvalues of the model written in reduced variables. Thus, we can observe that the satisfaction of the Blanchard and Kahn's conditions is not easier for one or the other kind of stability. When we go from the stability in the relative difference to the stability in the expanded difference we reduce the possibility of the existence of a solution path, but we also reduce the possibility of the existence of an infinity of solutions. Thus, if the Blanchard and Kahn's conditions are satisfied for both stabilities, the properties of existence and uniqueness are warranted for the case of the stability in the absolute difference, at last locally. In fact, there exists at least one solution stable in the absolute difference: the one which is stable in the expanded difference. However, if there existed more than one solution stable in the absolute differences, *a fortiori*, there would be several solutions stable in the relative difference. But, this is impossible because Blanchard and Kahn’s conditions are satisfied for this stability.

Cadiou, Dées and Laffargue (2000) investigate the case of a dynamic neo-classical growth model with vintage capital. When the relative risk aversion of consumers is low enough, the Blanchard and Kahn's conditions are satisfied by the linear approximation of the model written with reduced and expanded variables. However, when risk aversion is high enough, the Blanchard and Kahn's conditions are still satisfied by the linear approximation of the model written in reduced variables. But, the number of eigenvalues larger than one exceeds the number of lead variables by one when the linear approximation of the model is written with expanded variables. Thus, the first linear approximation defines a unique path for the economy. But, the second linear approximation has no solution. The simulation of the model written with its original variables may give a unique path for the economy in some situations or may have no solution in other cases[56].

An interest of the stability in the relative difference is that hysteresis is characterised by eigenvalues with absolute values equal to 1. We will still have to check that the number of associated eigen-vectors is equal to the order of multiplicity of the unitary eigenvalues, and are orthogonal to the right-hand side of the model written as equation (3). Then, we can apply Proposition 2. In the case of the stability in the expanded difference, we can apply Proposition 1, but without taking into account the eigenvalues with absolute values equal to $g_{\max}$.

Hysteresis raises a new problem at the level of the simulation of the model. In this case, the balanced growth path is no more unique, and we have an infinity of available terminal conditions for some of the anticipated variables. In general, only one of these terminal conditions is compatible with the initial conditions of the economy, but we do not know which. We can overcome this difficulty at the level of the writing of the model. Let us assume, for example, that the model builder considers as a possibility an hysteresis on the prices level, but not on the inflation rate. This means that the balanced growth path of prices is undetermined, but not the one of the inflation rate. Then, we can substitute all the led prices variables by the product of their current values by led inflation rates. By selecting at random a balanced growth path among all those which are possible, we do not introduce any error on the terminal conditions of the anticipated variables which are present in the model, and we can use the standard simulation algorithms quoted in the introduction.

I.3 Conclusion

In this paper we have explained how we can use the local conditions of Blanchard and Kahn to investigate the existence and the uniqueness of the solution of macro-econometric models of large size. To do that we have had to overcome the following difficulties: The model is non linear, its linear approximation gives coefficients which change over time, in the long run many variables grow at positive rates which differ between them, and finally the model may present an hysteresis. We have introduced the notion of stability in the absolute difference, which is the most natural but which does not allow the application of Blanchard and Kahn's conditions. Then, we have defined two other notions of stability which are consistent with the application

[56] We have met a situation where the linear approximation of the model written in reduced variables has less eigenvalues larger than 1 than lead variables, and where the linear approximation of the model written in expanded variables has more eigenvalues larger than 1 than lead variables. Then, in some situations we could compute a unique solution path for the model written with its original variables, in other cases there did not exist any solution and sometimes there existed an infinity of them.

of the results by Blanchard and Kahn: The stability in the relative difference and the stability in the expanded difference. If the Blanchard and Kahn's conditions are satisfied for these two stabilities, then the model has a unique solution under the condition of stability in the absolute difference.

Appendix

We consider the case where there does not exist any static variable and we use the notations of section 1. The model written in reduced variables is

(A1) $C_1 g_1^{-1} y_{t-1}^1 + (C_0^2 \big| C_0^1)\begin{bmatrix} y_t^2 \\ y_t^1 \end{bmatrix} + C_{-1} g_2 y_{t+1}^2 = 0$,

where g_1 et g_2 represent the diagonal matrices of the balanced growth rates of respectively the predetermined and the anticipated variables. Let us put:

(A2) $x_t^1 = y_t^1$, $x_t^2 = y_{t+1}^2$, $x_t = \begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix}$

Then, the model in reduced variables can be written:

(A3) $(C_1 g_1^{-1} \big| C_0^2) x_{t-1} + (C_0^1 \big| C_{-1} g_2) x_t = 0$

The model in expanded variables is:

(A4) $C_1 g_{\max} g_1^{-1} y_{t-1}^1 + (C_0^2 \big| C_0^1)\begin{bmatrix} y_t^2 \\ y_t^1 \end{bmatrix} + C_{-1} g_{\max}^{-1} g_2 y_{t+1}^2 = 0$

It can be written:

(A5) $(C_1 g_{\max} g_1^{-1} \big| C_0^2) x_{t-1} + (C_0^1 \big| C_{-1} g_{\max}^{-1} g_2) x_t = 0$

Let us denote by l an eigenvalue of system (A3) and by $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ the associated eigen-vector, the two components of which respectively correspond to x_t^1 et x_t^2. Then, we can easily show that we can associate to l the eigenvalue $g_{\max} l$ and the eigen-vector $\begin{bmatrix} V_1 \\ g_{\max} V_2 \end{bmatrix}$ of system (A5).

II STABILITY AND SIMULATION METHODOLOGY OF MARMOTTE

This section presents the simulation methodology of Marmotte, which includes first the creation of the analogous steady-state version of the model, then the calculation of the adjustment factors and the application of the stability conditions test, finally the computation of the solution path.

II.1 Steady State version of Marmotte and the calculation of the adjustment factors.

For the simulations of Marmotte the problem to be solved is a standard two-point boundary values one, accommodating initial and terminal conditions. The initial conditions are determined by the actual data whereas the terminal conditions, i.e. the values of the anticipated variables at the end of the simulation period, are defined using an analogous steady – state with growth model.

Like all perfect foresight models, Marmotte, can be expressed in the following compact form :

(1) $f(y_t, y_{t+1}, y_{t-1}, x_t) = 0$

where f(. .) - is a vector of 'n' equations,

y_t - is a vector of 'n' endogenous variables,

x_t - is a vector of 'm' exogenous variables.

Then the steady state version of the model (1) can be written in the following form :

(2) $f(y^{ss}, g\, y^{ss}, g^{-1}y^{ss}, x^{ss}) = 0$

where *g* is a diagonal matrix with dimension *n*. The diagonal of *g* contains the long run growth rates plus one of the endogenous variables of the model. The model (2) is cleared from any dynamics and it represents the path of the economy in the long run.

Marmotte, among others, is intended to be used for the assessment of the effects of different economic shocks vis-à-vis an economy without shocks. So, before trying to simulate different scenarios, one has to insure that in a situation without shocks, the model solutions correspond exactly to the actual data and forecasts. This trivial solution is termed the baseline solution. In case of Marmotte, the baseline simulation should be done *via* the inclusion of residuals in each equation, because part of the model is estimated and part of it is calibrated on the actual data. So, in order for the dynamic model to perfectly fit the actual data and the forecasts, one has to calculate the adjustment factors (or the residuals). The calculation of the residuals is not only important to "center" the model on the actual data and forecasts, but it also simplifies the task for the simulation of different scenarios.

To calculate the adjustment factors, the dynamic model is inverted, in such a way that the residuals become endogenous, all the other variables being fixed to their baseline values. This procedure ensures that the solution of the model (1) with the residuals exogenous, will exactly correspond to the baseline values. Then, it is necessary to check for the compatibility between model (1) and its steady state version. To do so, one should check that, when the baseline has reached its steady state path (after 2060), the solution of model (2) exactly gives the baseline using the calculated residuals from the dynamic model. [57]

Finally, starting and terminal conditions are necessary to simulate the dynamic model. The starting conditions are equal to the actual data, whereas the terminal conditions can be defined through the simulation of the steady state model. So, if the shock has permanent effects on the economy, the terminal conditions are defined by the simulation of the shock using the steady state model, otherwise the baseline provides the terminal conditions.

II.2 Stability Test

Some stability conditions should be satisfied before simulating the desired scenarios with Marmotte. To check the stability of Marmotte, the methodology of paragraph I-2 of this chapter is followed. Two versions of the linear approximation of the dynamic model are thus considered, in which all the variables are put on a common trend.

The first version is written in reduced variables, where the common trend has zero growth rate. The second version of the model is written in expanded variables, the growth rate of the common trend being the highest growth rate present in the model.

A sufficient condition for the existence and uniqueness of a solution path of the model (1) is that the Blanchard and Khan conditions are satisfied for the linear approximation of the model respectively written in reduced and in expanded variables. The eigenvalues of the second approximation are equal to those of the first, time the highest balanced growth rate present in the model.

The computation of the eigenvalues of Marmotte is currently impossible for computer memory reasons. Instead, the procedure described above was applied to sub models of Marmotte. A necessary condition for the stability of the whole model is that all sub models are stable.

Stability tests for one sub model written in reduced variables (including United States, Germany, France, Italy, Spain and Netherlands) give the following results: there are 881 eigenvalues with modulus greater than 1 (in absolute value) corresponding exactly to 881 lead variables in the model; there is one eigenvalue equal to 1 which corresponds to the hysteresis in the nominal exchange rate Euro/US$, and all the other eigenvalues are with modulus less than 1 (in absolute value). So, the stability of the model written in reduced variables is satisfied. The eigenvalues in the neighbourhood of 1 have the following values 1.066, 1.033, 1.000, 0.9613, 0.9602, 0.9600. Since the highest eigenvalue closest to 1 (from below) augmented by the highest balanced growth path rate in the model (which equals 0.03), is still less than one, it can be argued that the stability of the expanded version of the sub-model is also satisfied.

Besides, another useful indicator is the T^{th} power of the highest eigenvalue closest to 1 (from below), where T corresponds the number of the simulation years. It indicates the maximum “shift” of the variables linked with this eigenvalue likely to be observed at the end of the simulation period. It is clear that the longer the simulation period the lower that shift. In our case the period of simulation T is equal to 70 years, so that the shift at the end will be in the interval ±6.3%.

II.3 Simulation method

[57] One could also calculate the residuals from the steady state version of Marmotte, then check that the solution of the dynamic model including these residuals corresponds to the baseline.

To solve nonlinear models with rational expectations, the Troll software uses the Newton-Raphson combined with Laffargue-Boucekkine-Juillard (L-B-J) algorithm. The Newton-Raphson method is an iterative one. At the beginning, the system (1) above is linearised around the starting values of the endogenous variables and it is solved to determine the new set of endogenous variables. Then it is again linearised around this new set and it is solved. It continues likewise until the values of the last set of endogenous variables solution do not change significantly (according to some convergence criterion) from the previous iteration solution values.

The solution of the linearised system during each iteration involves computing the Jacobian matrix of system (1). A difficulty arises with this method, because the Jacobian matrix is of considerable size in large model with multiple leads and lags. All equations have indeed to be solve simultaneously for all periods at once, according to the following equation.

(3) $y^s = y^{s-1} - [\delta f/\delta y]^{-1} f(\ldots)$

where $[\delta f/\delta y]^{-1}$ is the inverse of the Jacobian matrix.

The main advantage of the LBJ algorithm is to avoid solving the whole system y^s . It seeks the triangularisation of the Jacobian and solves iteratively for each element of y^s either backwards or forwards depending on whether the Jacobian is transformed to an upper or rather a lower triangular matrix. There is thus no need to inverse the Jacobian and to store it in memory, a matrix of size [n*(T+2) x n*(T+2)], where *n* is the total number of equations and T the total number of the simulation periods. In case of Marmotte the size of the whole Jacobian is [80172 x 80172]. Using the L.B.J. method reduces the size of the matrix to be stored; it is then of order [n*T x TNLV], where TNLV is the total number of leads. For Marmotte, the matrix is of dimensions [80172 x 2566], which is still too much for the RAM memory of our computer. So, to solve the whole model, Marmotte has been divided into three sub models, and its simulation is done *via* three types of iterations.

The consistent single loop Newton Raphson method is used to solve each sub model, representing a type I iteration. A typical type II iteration represents a complete simulation of the three sub models one after the other. The rule is that during the simulation of a given sub model, the variables corresponding to the countries in the other groups, such as trade variables and competitors prices on foreign markets, are exogenous. A type II iteration encompasses thus three iterations of type 1, each of them corresponding to the solution of one submodel.

As an illustration, for the s^{th} iteration of the II type and second iteration of I type (second group of countries), the solution for the sub model G_2 is found by the following formula :

(4) $y^s_{G2} = f_1(y^{s-1}_{G2}, y^{s-2}_{G2}) - [\delta f/\delta y]^{-1} f(y^{s-1}_{G2}, y^{s-2}_{G2}, yp^s_{G1}, yp^{s-1}_{G1}, yp^{s-1}_{G3}, yp^{s-2}_{G3}, x_t, x_{t-1})$

where y^{s-1}_{G2} - is the solution for the sub model G_2 corresponding to the iteration s-1,

yp^s_{G1} - represents the solution during the iteration s for the endogenous variables of the sub model G_1 which appear (as exogenous) in the sub model G_2 ,

yp^{s-1}_{G3} - represents the solution during the iteration s-1 for the endogenous variables of the sub model G_3 which appear (as exogenous) in the sub model G_2 .

Equation (4) differs from the usual Newton Raphson formula (3). In equation (3), only the previous solution of type II iteration y^{s-1} is used to solve for y^s, whereas in formula (4), the solution is found by a Gauss Seidel first order method with a dampening factor: it includes a weighted average of y^{s-1} and y^{s-2}. At the end of each iteration of type II, a convergence error is calculated, whose distance from the convergence criterion will condition the resort to the dampening factor. If the comparison implies that the model is close to converge, the dampening is not used and the formula solving the sub models is the usual Newton Raphson one.

Once the convergence of II type iterations has been achieved, an extra type III iteration is necessary to ensure the convergence of the whole model. During iteration of type I and II, two variables of the model (1) are actually exogeneised (in order to have a well-defined Jacobian), namely the integer part of the expected lifetime of the new capital units (*TY*) and of the scrapping age of the oldest capital units (*AY*). However, the value of these two variables may change during the iteration of type I and II. If this is the case, the corresponding of *AY* and *TY* have to be changed at the beginning of the type III iteration. The model has converged when all types of iterations converge one after the other.

II.4 Particularities in the simulation of Marmotte

At the heart of Marmotte is a vintage capital supply block, which is written in discrete time with intra-period approximations. The supply block, among others, contains five first order conditions (FOC's) of two discounted sum functions with respect to five endogenous variables of the model and two equations, which define the aggregate supply and aggregate employment of the economy.

So, while simulating the model, one has to ensure that the FOC's are verified by the solutions of the model. The first discounted sum function is the value of the new units of capital introduced in the production process during the period *t* (which is equal to the present value of future expected profits of that unit minus the firing costs at the expected scrapping age) , and the second one is the value of the oldest units of capital still in operation during that period. In Marmotte, the above functions are optimised with respect to the following endogenous variables: the expected lifetime of the capital units newly introduced in the production process (*T*), its integer part (*TY*), the age of the oldest capital units still in operation during the *t* period (*A*), its integer part (*AY*) and the capital intensity of the new capital units (k_t). In addition, aggregate supply and aggregate employment in the economy depend on the optimised values of *AY*, *A* and $\boldsymbol{k}_t$.

The integer part of the expected lifetime of the new capital units is defined by the following inequalities :

$$\Psi_t\left(TY_t, T_t, \boldsymbol{k}_t\right) - \Psi_t\left(TY_t - 1, T_t, \boldsymbol{k}_t\right) > 0$$
$$\Psi_t\left(TY_t + 1, T_t, \boldsymbol{k}_t\right) - \Psi_t\left(TY_t, T_t, \boldsymbol{k}_t\right) < 0$$

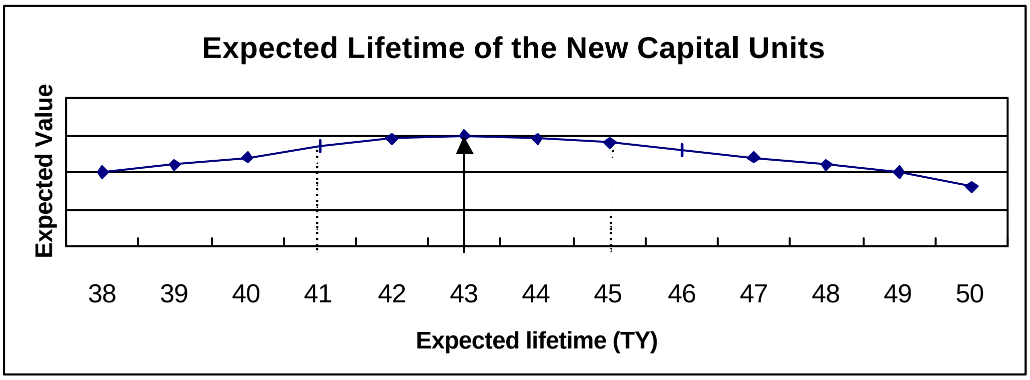
where function Ψ(..) represents the present value of future expected profits of the new units excluding the firing costs at the expected scrapping age. The intersection of that function with the space [TY, Ψ(.)] represents a two dimensional curve. In the neighbourhood of the steady state and for small growth rates, this curve can be shown to be smooth and such that the optimal value of *TY* (according to the inequality above) is unique and satisfies the following equation : *TY*= Integer(*T*). As an illustrative example, this curve might be similar to the one in the following graph, where the solution of the inequalities above corresponds to a maximising value of *TY* equal to 43.

The determination of *AY* can be done in the same fashion as for *TY*, i.e. using the same form of inequalities and the same functional form, Ψ(..). The only way to write the expressions of *TY* and *AY* was to employ a combination of special Troll functions "*SUM*" and "*SIGN*" as follows:

```
(5)TY = SUM(i=1 to 70: (1/2)*(1+SIGN((1-TCR(i))* ((SS_GGDP/SS_POP)**
(-i-(ΔT))*YQI-WPRRA(i)*((SS_GGDP/SS_POP)**(-ΔT))))+((INFLE(i)*(1-delta)/
1+(IS(i)+jj0(i))/100))**(ΔT))* (ΔT)*(1-CR(i+1))*((SS_GGDP/SS_POP)**
(-i-(ΔT))*YQI-(WPRRA(i) **(1-(ΔT))* WPRRA(i+1)**(ΔT)))-(XF(i)**(1-(ΔT))*
XF(i+1)**(ΔT)))-((INFLE(i-1)*(1-delta)/ (1+(IS(i-1)+jj0(i-1))/ 100))**
((ΔT)-1))*((ΔT)*(1-TCR(i))*((SS_GGDP/SS_POP)**(-i-(ΔT))*YQI-((WPRRA(i-
1))**(1-(ΔT))*(WPRRA(i)) **(ΔT))/(SS_GGDP/SS_POP))-((XF(i-1))**(1-
(ΔT))*(XF(i))**(ΔT))/(SS_GGDP / SS_POP))))) + RES_TY,
```

```
(6) AY = SUM(i=1 to 71:(1/2)*(1+SIGN((1-TCR(-1)) * ((SS_GGDP/SS_POP)**
(-i-(ΔA)+1)*YQI(-i)-WPRRA(-1)* ((SS_GGDP/SS_POP)**(-(ΔA))))+((INFL*
(1-delta)/(1+(IS(-1)+jj0(-1))/100))**(ΔA))*((ΔA)*(1-TCR)*((SS_GGDP/
SS_POP)**(-i-(ΔA)+1)*YQI(-i)-(WPRRA(-1)**(1-(ΔA)) *WPRRA**(ΔA)))-(XF(-
1)**(1-(ΔA))*XF ** (ΔA)))-((INFL(-1)* (1-delta)/(1+(IS(-2)+jj0(-
2))/100))**((ΔA)-1))*((ΔA)* (1-TCR(-1))*((SS_GGDP/SS_POP)**
(-i-(ΔA)+1) * YQI(-i)-(WPRRA(-2)**(1-(ΔA))*WPRRA(-1)**(ΔA))/ (SS_GGDP /
SS_POP))-(XF(-2)**(1-(ΔA))* XF(-1)**(ΔA))/(SS_GGDP/
SS_POP))))) +RES_AY,
```

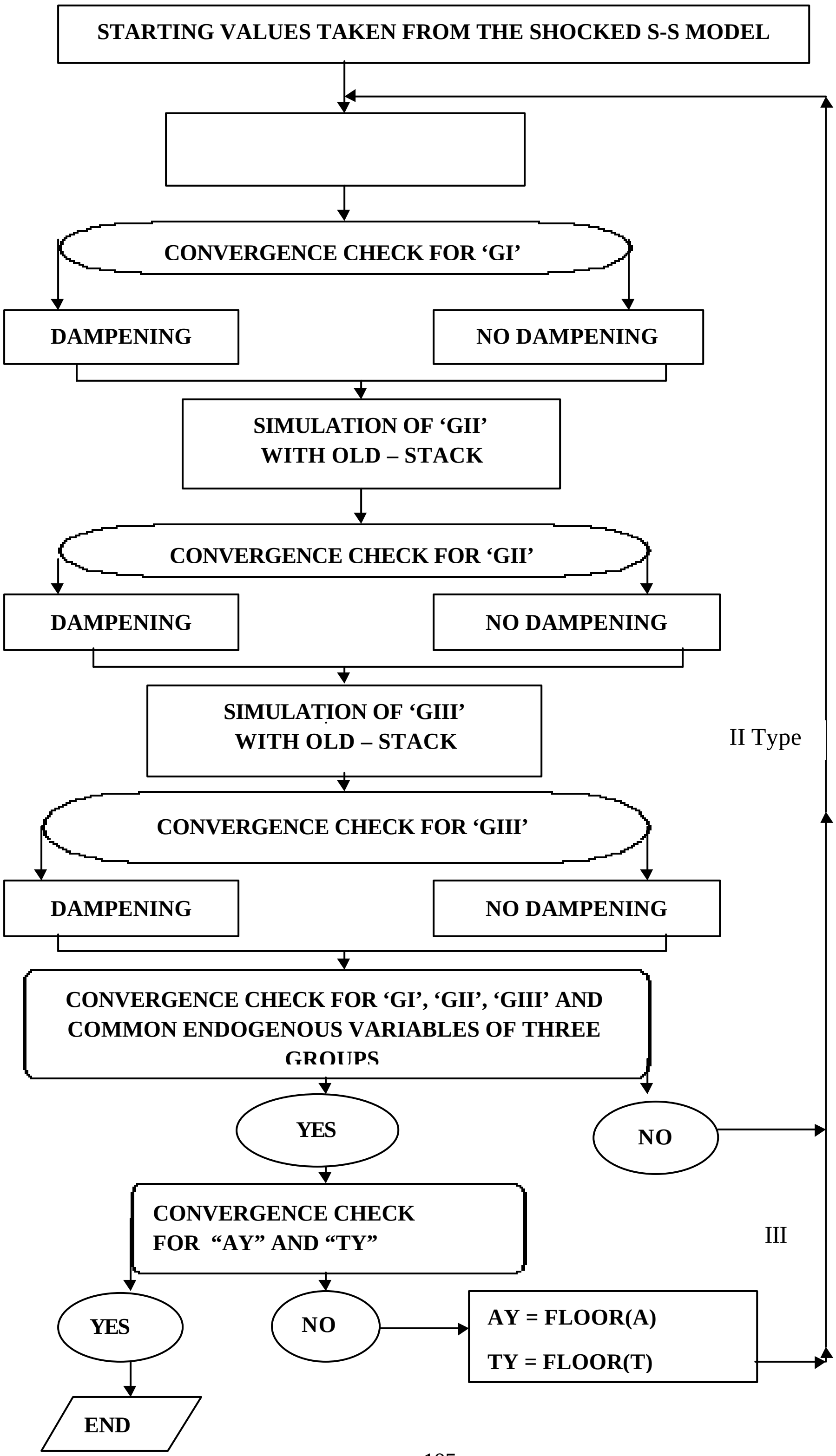
where `ΔT = T - TY`, `ΔA = A - AY` and `sign(n)=1` for `n>0` and `sign(n)=-1` for `n<0`.



The *TY* variable is determined by iterating over *i* (the sum index) in the following way. For each *i*, it is checked whether the first inequality is satisfied, i.e. $\Psi(i+1)-\Psi(i)>0$. It continues until the first inequality is no more verified for $i=i'$. The value of *i'* then determines the value of *TY*. As before, in the neighbourhood of the steady state and for small growth rate, the $\Psi(TY)$ function can be argued to be monotonic with a unique maximum in *TY*. In the same fashion, the solution for *AY* would be the one which maximises Ψ(*AY*).

If after the convergence of type II, *AY* and/or *TY* change (i.e. a change of *T* and/or *A* greater than one unit), then a different dynamic model should be generated. To avoid this difficulty, instead of using the calibrated values of *AY* and *TY* in the sum operators (indexed on *AY* and *TY*, see for example equation 54 in Final Appendix 2), a vector *Y2* centred on these values ± 2 is created. For example, for *TY*=43, the corresponding vector *Y2* is [41,42,43,44,45]. If the change in *TY* is less than 3 in absolute value, then they are put equal to the integer part of the new values of *T*, and new iterations of type I and II can start. This induces some flexibility in the simulation of Marmotte to the extent that some extra terms set to zero during iterations I and II are only included in the sum operators. However, if the change in *TY* falls outside the *Y2* vector, then another model with a larger vector *Y2* should be generated. In principle, a wider interval could increase the chances of convergence, but it would also increase the time until convergence and the memory requirements.

Appendix 1 : Macro scheme for the simulation of MARMOTTE



CHAPTER V

BASIC SIMULATIONS OF MARMOTTE

In Europe, the autonomy and the co-ordination of national fiscal policies, the role of a common monetary policy and of the exchange rate system implied by EMU have important consequences on the international transmission of economic shocks. Marmotte offers both a careful description of the EU countries, including significant structural asymmetries and a comprehensive description of interactions between EU members as well as with other countries in the world, through trade and capital flows. It is thus especially well adapted to investigate the effects of asymmetric shocks (originating in a specific European country), symmetric shocks (originating outside the EU), and asymmetries in the economic structures of European countries.

The main purpose of this chapter is to assess the differentiated responses of European countries to standard shocks: demand and supply shocks. The demand shock is simulated through a permanent increase in government expenditures by 1% of the GDP. The supply shock corresponds to a productivity shock that rises permanently output by 1% in the long run. Furthermore, we investigate the effects of these shocks on Europe with respect to their symmetric or asymmetric nature, i.e. by distinguishing their origin. A shock occurring in the United States stands for a symmetrical shock on Europe, whereas if it occurs in a country of the euro area, in Germany, it represents an asymmetric shock. Four simulations are thus computed.

We are especially interested in assessing the effects of these shocks on the US and Germany (the countries where shocks occur), France and on the UK (where monetary policy is defined at a national level). The simulation results can be given clear and precise interpretations. They are expressed in percentage deviations from the baseline scenario, except for the real and nominal interest rates which are given in percentage points. In addition to the effects of the different shocks over a period of 10 years, all the tables give also the very long-run effects, computed with the steady state version of Marmotte.

I THE EFFECTS OF DEMAND SHOCKS

Simulation results of shocks in government expenditures are first considered. Government budget includes an item, called other taxes, which is the difference between lump sum taxes and lump sum transfers. These taxes are non-distortionary and are used as a device to stabilise public debt and prevent Government from entering a *Ponzi* finance process. They are modelled as a fiscal policy rule that implies an increase in these taxes when public debt is larger than the target set by the Government, which warrants the inter temporal solvency of Governments.

I.1 Shock in the United States Government expenditures (increase by 1% of GDP)

In the country where the shock occurs, the United States, the increase in Government expenditures has strong crowding-out effect on the other demand components (Table 1).

The demand shock has inflationary effects, as demand tends to be higher than supply, which is sticky in the short-run. The price level increases progressively as compared to the baseline scenario. After 10 years, the output deflator increases by 0.24 %. However, the stickiness of price induces a Keynesian effect on the output of the private sector.[58] It increases by 0.19% in the first year, and it stays at a higher level than in the reference path for three years. As private output is about equal to 2/3 of GDP, the short run Keynesian multiplier is about equal to 0.12.

Monetary policy reacts to price increase by raising the short-term interest rate: by 6 basis point for the first year and by a decreasing amount over the 8 following years. This leads to a slight increase in the real interest rate that contributes to depress the other components of demand, especially investment.[59]

Households' consumption decreases strongly in the US as they expect a future increase in taxes to finance the government expenditures. It decreases by 0.27 % during the year of the shock. After 10 years, consumption decrease reaches 1.25 %. The increase in Government expenditures will lead in the future to an increase in lump-sum taxes. Thus public balance, after deteriorating by 0.84 % of GDP in the short run, stabilises slowly over time.

An expected increase in the cost of capital in the present and the future, associated with an expected decrease in labour cost, make firms immediately reduce their investment by 1.98 %. After 10 years, investment is still below the reference scenario (by -0.20 %). The reduction in capital also implies a reduction in labour demand. Even if the real cost of labour decreases, the medium-run effect on employment is negative: employment is reduced by 0.10 % after 10 years. The decrease in labour cost relative to capital cost leads to implement new units of production that are more labour intensive. But, the strong reduction in investment and the relative complementary between labour and capital induce a negative effect on employment.

As Government consumption is mostly composed of national goods, its increase implies a higher relative price of American goods relatively to foreign goods. The real exchange rate (the ratio of imports price to the absorption deflator)[60] decreases by 0.43 % immediately. This appreciation reduces very slowly over time. As a consequence, the trade balance deteriorates.[61]

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[58] In ours results, production means the output of the private sector. It differs from GDP by the output of the public sector (measured by total wages pays by this sector), VAT and custom duties perceived by the government. All these various components are evaluated at different prices, changes in production and in GDP may thus differ at some extent.

[59] The information given by the variation in current investment is completed by the changes in the age of the oldest production units kept in activity.

[60] Actually, there are two import prices in Marmotte. The first appears in the imports price equations and in the imports equations where it is used to define the competitiveness of the country. The second includes a correction which keeps constant across variants the discrepancy between the world value of imports and the world value of exports, both computed in dollars. We have preferred to use the first imports price in the definition of the real exchange rate.

[61] The trade balances exclude imports and exports of primary goods. They are measured in current dollars, and their total over the 17 modelled countries and the rest of the world is exogenous. The control path of Marmotte reproduces OECD forecasts for the 5 first years, then converges to a balanced growth path over a period of more than one century. To make our simulations independent of the economic conditions prevailing at the beginning of the 21st century, we ran our simulations over the period 2130-2200. This choice must be understood before interpreting the variations of the balance of trade measured in billions of US dollars. In 2130, US GDP is evaluated to $ 63441 billions. It increases in nominal terms by 3% per year.

There is a J-curve effect since in the short run the trade balance turns a surplus. However, this positive effect vanishes rapidly. After 2 years, the effect on trade balance becomes negative.

In the other countries, the effects of the US demand shocks are significant in the short-medium run. These effects result from the real appreciation of the US dollar. The euro immediately depreciates by 0.95 % and the pound by 1.07 %. However, after 3 years, the depreciation of the pound is less than the euro's. Output deflators move little and in different directions in Germany, France and the United Kingdom. The cost of labour stay almost unchanged. However, if we except short-run positive Keynesian effects in a few countries such as France, production decreases everywhere: after 10 years by 0.04 % in Germany, and 0.02 % in the United Kingdom.

In spite of the real depreciation of European currencies, the effects on European trade balances, denominated in current US dollars, are negative. This can be explained by the surplus of the rest of the world resulting from higher US prices. Marmotte allocates this surplus between industrialised countries proportionally to the market shares of these countries in the exports of the rest of the world, so as to keep a world balance between the value of total exports and the value of total imports. This feature of Marmotte is probably unsatisfactory, and will require a better modelling of the rest of the world.

I.2 Shock in Germany government expenditures (increase by 1% of GDP)

Table 2 gives the results of the simulation of a demand shock in Germany. The short-run Keynesian effect is very high in Germany: output increases by 0.91 % in the year of the shock, which corresponds to a Keynesian multiplier of about 0.6. The difference of this result with the one we got for the US, is that Germany is related to countries of the euro zone by a fixed exchange rate system, when the US work under a flexible exchange rate system.[62]

In the following years, the positive effect on German output is much lower and decreasing. However, it is still positive after 10 years. This result is related to the real appreciation of the euro resulting from higher Government spending in Germany.

The main difference with respect to the previous simulation rests on the membership of Germany in the EMU: the inflationary response to a demand shock is much more pronounced in Germany than in the United States, because the ECB monetary policy does not react to German inflation only but to an average inflation computed at the European level.

Whereas US short term interest rates increased as a response to US inflation by 6 basis points, in Europe, the demand shock in Germany leads to a slighter increase (4 basis point), in spite of the higher German inflation. This is explained by the fact that prices decrease in some of the

Thus, the high variations in the balances of trade which can be noticed in the tables, are actually quite moderate. Maybe we should have measured the balances of trades in shares of American GDP, and not in billions of dollars.

[62] The well-known results of the Mundell and Fleming model (that is fiscal policy is efficient under a fixed exchange rate system, and inefficient under a flexible exchange rate system) are corroborated by our simulations.

other European country. For instance, in France prices move down immediately (by 0.06 %) whereas German prices move up (by 0.28 %).[63]

In France, for the first years of the simulation, output price decreases when euro appreciates. Then it increase and euro depreciates. The effect on the other economies is insignificant. The most significant effect concerns the depreciation of the pound. However, this depreciation is not very important (0.06 percent at the beginning of the simulation, 0.02 percent after 10 years).

In the first year of the simulation, when German output increases by much, the German balance of trade turns on a surplus. In years 2 and 3, German output almost comes back to its initial level, but the crowding out of private consumption is still incomplete: as prices stay temporarily under their long run level, because of their stickiness, households do not reduce their consumption as much as they will do later. Thus, the balance of trade is in deficit. Later on, the decrease in consumption leads to a fall of the domestic demand for import goods, which induces a surplus of the trade balance.

II THE EFFECTS OF SUPPLY SHOCKS

The simulation of a productivity shock raises specific difficulties in a model with a putty-clay technology. Here, a technology shock only affects new production units, without changing the productivity of old units. So, it diffuses progressively to the entire stock of capital. The goal of this simulation is to reach a permanent increase by 1% of output in the long-run. The simulation consists in an unanticipated and permanent increase in the total productivity term of the production function by a factor of adequate size[64]. Tables 3 and 4 display the results of productivity shocks in the United States and Germany, respectively.

II.1 Productivity shock in the United States that permanently rises output by 1%

As Table 3 shows, the investment response to a productivity shock is quick and high. Firms invest strongly in the short run (+2.73 % in year 1). The increase in investment is also largely significant in the medium run (+1.57 % after 10 years). In the long-run, the permanent increase in the stock of capital leads to an increase in investment by 0.15%.

The productivity shock increases the output deflator in the short and medium run: supply raises progressively, but demand (especially investment) increases immediately. US monetary policy adjusts to inflation by increasing the nominal interest rate. The upward pressure on interest rate slows the raise in investment and depresses consumption for about six years

[63] Whereas the ECB considers an harmonised consumer price index, in MARMOTTE, the prices entering the Taylor rules are production prices. However, the movements between consumer and production prices are quite similar.

[64] MARMOTTE assumes a CES production technology with an elasticity of substitution equal to 0.6. Our technological shock is Solow neutral. We also made simulations for Harrod neutral and Hicks neutral shocks (by changing the coefficient of labour or capital, in the production function). The results we got do not differ much from the ones given here. We also made simulations assuming that the supply shock increases the productivity of current investment *and* of all the old production units. We have chosen not to give the results of these simulations here.

Real labour cost increasing, labour becomes more expensive relatively to capital, so the newly implemented units of production become more capital intensive than the old ones. Employment rise just matches the need for labour induced by the new units of production. In the long run, both factors are more productive: investment increase by 0.15% and employment by 0.36 %, whereas output grows by 1%.

The higher supply of US goods decreases the relative price of the US good with respect to foreign goods, that is the US real exchange rate depreciates.[65] This is realised in part *via* a depreciation in the nominal exchange rate (the euro appreciates by 0.71% immediately and by 2.53% after 10 years).

In the short and medium run, the supply of American goods is constrained by the putty-clay technology: the productivity shock only affects the new production units. On the other hand, American investment is high, and this part of the world demand has a higher component in American commodities than other components. This, temporarily limits the real depreciation of the US dollar (0.32% in the first year and 0.62% in the tenth year)[66].

The appreciation of the euro entails a deflationary effect on the European production price level. In Germany, the decrease in prices starts two years after the shock (-0.01 %) and is significant after 10 years (-0.23 %). In France, we get similar results (-0.03 % in year 3 and -0.39 % in year 10). This has positive effects on the demand side. First, it pushes consumption up. The increase in the real cost of labour and the decrease in real interest rate strengthen the positive effect on both consumption and investment. The immediate effect on investment in Germany is a rise of 0.68% and after 10 years it is significant (+1.46 % in Germany and 1.04% in France). However, in the very long run, the US productivity shock has little effect on the European supply.

The balance of trade deteriorates in the US because of the strong increase in investment. It also deteriorates in European countries because of the real appreciation of the euro and the pound. As US production price increases, the balance of trade of the rest of the world improves.

II.2 Productivity shock in Germany that rises permanently output by 1%

Table 4 displays the results of this simulation. The effects on Germany are similar to the effects observed on the US in the previous simulation. French output increases significantly, but British and American output does not move. This asymmetry is due to the fact that France is linked to Germany by a fixed exchange rate, but Germany and the US (or the UK) are related by a flexible exchange rate system.

The real depreciation of the euro, increases the output deflator by a small amount in Germany (where supply increases) and by a higher level in France (where supply hardly moves). The ECB follows a more restrictive monetary policy than if it only considered German prices. The balance

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[65] MARMOTTE is a one sector model. Thus, it cannot generate a Balassa-Samuelson effect (a real appreciation of the US dollar). According to it, an increase in the productivity of the tradable good sector, increases the demand for non tradable goods, the supply of which is sticky, which appreciates the real exchange rate.

[66]An alternative simulation with identical content of import good for all the demand components shows that the real depreciation of the dollar is a robust result, albeit of a smaller size.

of trade deteriorates in Germany because of the raise in investment, and in spite of the real depreciation of the euro. We get the same result for France, but with less strength.

Table 1: Shock in the United States government expenditures (increase by 1% of Production)

United States	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**	**Y10**	**S-S**
Private Production	0,19	0,09	0,02	-0,03	-0,08	-0,12	-0,15	-0,17	-0,19	-0,20	0,06
Consumption	-0,27	-0,51	-0,69	-0,84	-0,96	-1,05	-1,12	-1,18	-1,22	-1,25	-1,18
Investment	-1,98	-1,49	-1,08	-0,75	-0,51	-0,34	-0,23	-0,15	-0,11	-0,09	0,14
Scrapping age	-0,01	-0,01	0,02	0,04	0,06	0,07	0,08	0,08	0,08	0,08	-0,01
Employment	-0,01	-0,07	-0,09	-0,11	-0,11	-0,11	-0,11	-0,11	-0,11	-0,10	0,01
Real wages	0,02	0,01	-0,03	-0,06	-0,08	-0,10	-0,11	-0,11	-0,11	-0,11	0,05
Production price	0,06	0,11	0,16	0,19	0,22	0,24	0,25	0,25	0,25	0,24	
Nominal exchange rate	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Real exchange rate	-0,43	-0,44	-0,43	-0,42	-0,40	-0,38	-0,36	-0,34	-0,32	-0,31	-0,05
Short-term interest rate*	0,06	0,06	0,05	0,04	0,03	0,03	0,02	0,01	0,00	0,00	
Real interest rate*	0,01	0,01	0,01	0,01	0,02	0,02	0,02	0,02	0,01	0,01	-0,01
Public balance to GDP*	-0,84	-0,79	-0,74	-0,69	-0,65	-0,60	-0,56	-0,53	-0,50	-0,47	-0,24
Trade balance*(Bill. of U	235	64	-80	-208	-317	-402	-461	-495	-503	-490	52
Germany											
Private Production	0,00	-0,04	-0,05	-0,06	-0,06	-0,06	-0,06	-0,05	-0,05	-0,04	-0,01
Consumption	0,05	0,08	0,09	0,10	0,10	0,10	0,10	0,10	0,10	0,10	-0,05
Investment	-0,43	-0,37	-0,18	-0,03	0,10	0,21	0,28	0,33	0,36	0,36	-0,01
Scrapping age	0,00	0,00	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,00	0,00
Employment	0,00	-0,01	-0,02	-0,03	-0,03	-0,03	-0,02	-0,02	-0,01	0,00	-0,01
Real wages	0,00	0,00	-0,01	-0,02	-0,02	-0,02	-0,02	-0,01	-0,01	0,00	0,00
Production price	0,00	-0,01	-0,01	-0,02	-0,02	-0,03	-0,04	-0,05	-0,06	-0,08	
Nominal exchange rate	0,95	0,90	0,86	0,81	0,76	0,71	0,66	0,62	0,58	0,54	
Real exchange rate	0,41	0,42	0,41	0,41	0,40	0,39	0,38	0,37	0,36	0,36	0,10
Short-term interest rate*	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,02	-0,02	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	-0,01	-0,02	-0,02	-0,02	-0,02	-0,01	-0,01	-0,01	0,00	0,00	0,00
Trade balance*(Bill. of U	-87	-164	-229	-288	-337	-374	-399	-414	-418	-414	-13
France											
Private Production	0,37	0,03	0,01	0,01	0,00	0,00	0,00	-0,01	0,00	0,00	0,00
Consumption	0,06	0,10	0,12	0,13	0,14	0,14	0,14	0,14	0,13	0,13	-0,03
Investment	0,91	-0,07	-0,02	0,06	0,14	0,20	0,25	0,28	0,29	0,28	0,00

Scrapping age	-0,02	0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,02	0,00
Employment	-0,01	0,03	0,02	0,01	0,01	0,01	0,02	0,02	0,03	0,03	0,00
Real wages	-0,03	0,00	0,02	0,01	0,01	0,01	0,01	0,01	0,02	0,02	0,00
Production price	0,11	0,11	0,10	0,10	0,09	0,09	0,08	0,06	0,05	0,03	
Nominal exchange rate	0,95	0,90	0,86	0,81	0,76	0,71	0,66	0,62	0,58	0,54	
Real exchange rate	0,26	0,26	0,27	0,26	0,26	0,26	0,25	0,25	0,24	0,24	0,06
Short-term interest rate*	-0,01	-0,01	-0,01	-0,01	-0,01	-0,02	-0,02	-0,02	-0,02	-0,02	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,01	0,00
Public balance to GDP*	-0,02	-0,01	0,00	-0,01	0,00	0,00	0,00	0,00	0,01	0,01	0,00
Trade balance*(Bill. of U	-49	-117	-145	-170	-193	-212	-226	-234	-237	-235	-10

United Kingdom	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	S-S
Private Production	-0,02	-0,03	-0,04	-0,04	-0,04	-0,04	-0,04	-0,03	-0,03	-0,02	-0,01
Consumption	0,02	0,03	0,05	0,06	0,07	0,08	0,08	0,09	0,09	0,10	-0,06
Investment	-0,23	-0,19	-0,11	-0,03	0,06	0,13	0,19	0,24	0,27	0,28	-0,01
Scrapping age	0,00	0,00	0,00	0,00	0,00	0,01	0,00	0,00	0,00	0,00	0,00
Employment	0,00	-0,01	-0,02	-0,02	-0,02	-0,02	-0,02	-0,01	-0,01	0,00	-0,01
Real wages	0,00	0,01	0,00	0,00	-0,01	-0,01	-0,01	0,00	0,00	0,00	0,00
Production price	0,00	-0,01	-0,01	-0,02	-0,02	-0,03	-0,03	-0,04	-0,05	-0,05	
Nominal exchange rate	1,07	0,98	0,88	0,79	0,71	0,64	0,58	0,53	0,49	0,46	
Real exchange rate	0,51	0,47	0,42	0,38	0,35	0,32	0,30	0,28	0,27	0,26	0,10
Short-term interest rate*	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	-0,01	-0,01	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00
Trade balance*(Bill. of U	-65	-85	-106	-130	-152	-171	-188	-199	-207	-210	5

Note: % deviation from baseline, except * (absolute differences from base).

Table 2: Shock in Germany government expenditures (increase by 1% of Production)

United States	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	S-S
Private Production	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01
Consumption	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01	-0,01
Investment	-0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,02
Scrapping age	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Employment	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Real wages	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01
Production price	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Nominal exchange rate	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Real exchange rate	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01
Short-term interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	

Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Trade balance*(Bill. of U	5,2	-2,7	-5,0	-4,8	-3,3	-1,4	0,2	1,2	1,6	1,5	-3,7
Germany											
Private Production	0,91	0,15	0,12	0,11	0,10	0,09	0,08	0,07	0,06	0,05	0,05
Consumption	-0,67	-1,09	-1,36	-1,52	-1,62	-1,68	-1,72	-1,75	-1,76	-1,77	-1,74
Investment	0,58	-0,83	-0,49	-0,24	-0,09	0,00	0,04	0,06	0,07	0,07	0,05
Scrapping age	-0,02	-0,02	-0,02	-0,01	0,00	0,00	0,00	0,01	0,01	0,01	0,00
Employment	-0,01	0,01	-0,02	-0,03	-0,03	-0,03	-0,03	-0,02	-0,02	-0,02	0,05
Real wages	-0,04	0,03	0,03	0,02	0,01	0,00	0,00	-0,01	-0,01	-0,01	0,00
Production price	0,28	0,32	0,36	0,40	0,45	0,49	0,52	0,56	0,58	0,61	
Nominal exchange rate	-0,03	-0,01	0,02	0,04	0,07	0,09	0,11	0,13	0,15	0,16	
Real exchange rate	-0,27	-0,28	-0,29	-0,31	-0,32	-0,33	-0,35	-0,36	-0,37	-0,37	-0,38
Short-term interest rate*	0,04	0,04	0,05	0,05	0,04	0,04	0,03	0,03	0,03	0,02	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	-1,06	-0,98	-0,93	-0,88	-0,83	-0,78	-0,73	-0,68	-0,64	-0,60	-0,29
Trade balance*(Bill. of U	63,8	-30,1	-0,8	21,7	35,8	43,8	47,9	49,6	49,8	49,3	18,8

France	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**	**Y10**	**S-S**
Private Production	-0,21	0,02	0,05	0,05	0,05	0,04	0,04	0,03	0,03	0,03	-0,005
Consumption	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,00	0,00	-0,03
Investment	-0,76	-0,04	0,07	0,09	0,10	0,10	0,09	0,08	0,07	0,06	-0,01
Scrapping age	0,01	-0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Employment	0,00	-0,04	-0,03	-0,02	-0,02	-0,02	-0,02	-0,01	-0,01	-0,01	-0,01
Real wages	0,03	0,02	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00
Production price	-0,06	-0,04	-0,02	0,01	0,03	0,05	0,07	0,08	0,09	0,10	
Nominal exchange rate	-0,03	-0,01	0,02	0,04	0,07	0,09	0,11	0,13	0,15	0,16	
Real exchange rate	0,06	0,06	0,06	0,06	0,07	0,07	0,07	0,08	0,08	0,08	0,07
Short-term interest rate*	0,02	0,03	0,03	0,02	0,02	0,02	0,01	0,01	0,01	0,01	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	0,00	-0,01	-0,02	-0,01	-0,01	-0,01	-0,01	-0,01	0,00	0,00	0,00
Trade balance*(Bill. of U	-10,2	11,5	8,0	2,4	-2,4	-6,0	-8,3	-9,6	-10,1	-10,1	-1,3
United Kingdom											
Private Production	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01
Consumption	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,02	-0,02	-0,02	-0,02	-0,01
Investment	0,01	0,01	0,01	0,00	-0,01	-0,01	-0,02	-0,02	-0,02	-0,02	-0,01
Scrapping age	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Employment	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	-0,01
Real wages	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Production price	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Nominal exchange rate	0,06	0,06	0,05	0,05	0,04	0,03	0,03	0,02	0,02	0,02	
Real exchange rate	0,08	0,07	0,07	0,06	0,06	0,05	0,05	0,05	0,04	0,04	0,05

Short-term interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Trade balance*(Bill. of U	-12,2	-11,9	-8,9	-5,1	-1,3	1,9	4,3	5,7	6,3	6,1	-3,2

Note: % deviation from baseline, except * (absolute differences from base).

Table 3: Productivity shock in the United States that increases Production permanently by 1%

United States	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	S-S
Private Production	0,58	0,56	0,54	0,53	0,53	0,54	0,56	0,59	0,62	0,67	1,00
Consumption	-0,04	-0,02	0,01	0,03	0,07	0,10	0,14	0,18	0,22	0,26	1,07
Investment	2,73	2,81	2,78	2,66	2,50	2,30	2,10	1,90	1,72	1,57	0,15
Scrapping age	-0,04	-0,05	-0,09	-0,13	-0,17	-0,21	-0,25	-0,29	-0,32	-0,36	-0,06
Employment	-0,02	0,07	0,15	0,22	0,28	0,33	0,38	0,41	0,44	0,46	0,36
Real wages	0,05	0,09	0,15	0,20	0,26	0,32	0,37	0,42	0,47	0,52	0,71
Production price	0,18	0,30	0,38	0,42	0,42	0,39	0,34	0,28	0,20	0,13	
Nominal exchange rate	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Real exchange rate	0,32	0,31	0,32	0,34	0,38	0,42	0,47	0,52	0,57	0,62	0,59
Short-term interest rate*	0,24	0,19	0,14	0,09	0,06	0,03	0,01	0,00	-0,01	-0,02	
Real interest rate*	0,12	0,11	0,10	0,09	0,09	0,08	0,07	0,07	0,06	0,06	0,06
Public balance to GDP*	-0,10	-0,05	-0,01	0,02	0,05	0,07	0,09	0,10	0,11	0,11	0,05
Trade balance*(Bill. of U!	-546	-870	-1096	-1216	-1237	-1174	-1043	-862	-647	-415	44
Germany											
Private Production	0,01	0,01	0,03	0,06	0,10	0,13	0,17	0,20	0,23	0,26	0,08
Consumption	0,28	0,45	0,56	0,63	0,67	0,69	0,71	0,72	0,72	0,72	0,79
Investment	0,68	1,02	1,33	1,58	1,74	1,81	1,80	1,72	1,60	1,46	0,08
Scrapping age	0,00	0,00	0,00	-0,01	-0,03	-0,04	-0,06	-0,07	-0,09	-0,10	0,00
Employment	0,00	0,02	0,04	0,07	0,10	0,14	0,18	0,22	0,26	0,29	0,08
Real wages	-0,01	-0,01	0,00	0,02	0,03	0,05	0,07	0,09	0,11	0,13	0,00
Production price	0,00	0,00	-0,01	-0,03	-0,04	-0,07	-0,10	-0,14	-0,18	-0,23	
Nominal exchange rate	-0,71	-1,02	-1,31	-1,57	-1,79	-1,99	-2,15	-2,30	-2,42	-2,53	
Real exchange rate	-0,29	-0,41	-0,52	-0,62	-0,73	-0,82	-0,91	-0,99	-1,06	-1,12	-1,09
Short-term interest rate*	-0,02	-0,02	-0,03	-0,03	-0,04	-0,05	-0,06	-0,06	-0,07	-0,07	
Real interest rate*	-0,01	-0,01	-0,01	-0,01	-0,02	-0,02	-0,02	-0,02	-0,02	-0,02	0,00
Public balance to GDP*	0,05	0,07	0,09	0,10	0,12	0,14	0,15	0,16	0,17	0,18	0,04
Trade balance*(Bill. of U!	-202	-350	-440	-485	-492	-465	-412	-337	-245	-143	21
France											
Private Production	0,02	-0,01	-0,03	-0,03	-0,01	0,00	0,03	0,05	0,07	0,10	0,04
Consumption	0,18	0,30	0,38	0,44	0,47	0,50	0,51	0,52	0,53	0,53	0,64
Investment	0,64	0,87	1,06	1,22	1,32	1,36	1,34	1,27	1,17	1,04	0,04
Scrapping age	0,00	0,01	0,01	0,00	-0,01	-0,03	-0,04	-0,06	-0,08	-0,09	0,00
Employment	0,00	0,02	0,04	0,06	0,09	0,11	0,14	0,16	0,18	0,20	0,04
Real wages	-0,01	-0,02	-0,01	0,00	0,02	0,04	0,06	0,09	0,11	0,13	0,00
Production price	0,01	0,00	-0,03	-0,07	-0,11	-0,16	-0,21	-0,27	-0,33	-0,39	
Nominal exchange rate	-0,71	-1,02	-1,31	-1,57	-1,79	-1,99	-2,15	-2,30	-2,42	-2,53	
Real exchange rate	-0,26	-0,36	-0,44	-0,52	-0,60	-0,67	-0,73	-0,79	-0,84	-0,89	-0,90
Short-term interest rate*	-0,02	-0,04	-0,05	-0,06	-0,06	-0,07	-0,08	-0,08	-0,08	-0,08	
Real interest rate*	-0,01	-0,01	-0,01	-0,01	-0,01	-0,02	-0,02	-0,02	-0,02	-0,02	0,00
Public balance to GDP*	0,05	0,07	0,09	0,10	0,11	0,13	0,14	0,15	0,15	0,16	0,03
Trade balance*(Bill. of U!	-88	-176	-237	-271	-282	-272	-245	-205	-155	-97	-2

United Kingdom	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	S-S
Private Production	-0,08	-0,07	-0,04	-0,01	0,03	0,07	0,11	0,16	0,20	0,23	0,06
Consumption	0,06	0,10	0,14	0,17	0,19	0,20	0,22	0,23	0,23	0,23	0,31
Investment	0,48	0,88	1,19	1,40	1,53	1,59	1,58	1,53	1,44	1,33	0,06
Scrapping age	0,01	0,01	0,01	0,00	-0,01	-0,03	-0,04	-0,06	-0,08	-0,09	0,00
Employment	0,01	0,02	0,04	0,06	0,09	0,12	0,16	0,19	0,22	0,24	0,06
Real wages	-0,01	-0,02	-0,02	0,00	0,02	0,04	0,06	0,09	0,11	0,13	0,00
Production price	-0,02	-0,05	-0,08	-0,11	-0,14	-0,17	-0,21	-0,24	-0,28	-0,32	
Nominal exchange rate	-0,71	-0,99	-1,24	-1,45	-1,62	-1,76	-1,87	-1,96	-2,02	-2,08	
Real exchange rate	-0,24	-0,30	-0,36	-0,42	-0,46	-0,50	-0,54	-0,57	-0,59	-0,61	-0,71
Short-term interest rate*	-0,04	-0,04	-0,04	-0,05	-0,05	-0,05	-0,05	-0,05	-0,06	-0,06	
Real interest rate*	-0,01	-0,01	-0,01	-0,01	-0,01	-0,02	-0,02	-0,02	-0,02	-0,02	0,00
Public balance to GDP*	0,04	0,04	0,05	0,05	0,06	0,07	0,08	0,08	0,09	0,09	0,02
Trade balance*(Bill. of US	-137	-229	-293	-329	-342	-333	-305	-263	-209	-148	7

Note: % deviation from baseline, except * (absolute differences from base).

Table 4: Productivity shock in Germany that increases Production permanently by 1%

United States	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	S-S
Private Production	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01
Consumption	0,00	0,01	0,01	0,01	0,01	0,01	0,01	0,01	0,02	0,02	0,01
Investment	0,01	0,00	0,00	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	0,00	0,03
Scrapping age	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Employment	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Real wages	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01
Production price	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,01	
Nominal exchange rate	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Real exchange rate	0,00	-0,01	-0,01	-0,01	-0,02	-0,02	-0,02	-0,03	-0,03	-0,03	-0,02
Short-term interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Trade balance*(Bill. of US$	-2,2	-3,4	0,8	5,8	10,8	14,6	16,8	17,1	15,4	11,9	10,9
Germany											
Private Production	0,40	0,05	0,11	0,18	0,24	0,30	0,35	0,40	0,44	0,47	0,99
Consumption	-0,10	-0,13	-0,12	-0,10	-0,06	-0,02	0,03	0,08	0,13	0,17	0,85
Investment	1,48	0,48	0,57	0,66	0,72	0,76	0,78	0,80	0,81	0,81	0,79
Scrapping age	0,01	0,01	-0,03	-0,06	-0,09	-0,12	-0,14	-0,17	-0,19	-0,21	-0,09
Employment	0,01	0,07	0,07	0,08	0,09	0,10	0,11	0,12	0,13	0,14	0,28

Real wages	-0,04	0,01	0,06	0,11	0,14	0,18	0,22	0,25	0,28	0,31	0,82
Production price	0,12	0,10	0,09	0,09	0,10	0,11	0,13	0,15	0,17	0,18	
Nominal exchange rate	0,20	0,27	0,34	0,41	0,48	0,54	0,61	0,67	0,72	0,78	
Real exchange rate	0,03	0,10	0,17	0,23	0,28	0,33	0,37	0,40	0,44	0,47	0,90
Short-term interest rate*	0,08	0,08	0,08	0,09	0,09	0,09	0,08	0,08	0,08	0,07	
Real interest rate*	0,09	0,09	0,08	0,08	0,07	0,07	0,07	0,06	0,06	0,06	0,00
Public balance to GDP*	-0,09	-0,05	-0,04	-0,02	-0,01	0,00	0,02	0,03	0,04	0,05	0,07
Trade balance*(Bill. of US$	8,7	-64,0	-57,0	-49,2	-43,7	-40,6	-39,7	-40,8	-43,5	-47,6	-48,0

France	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**	**Y10**	**S-S**
Private Production	0,05	0,23	0,25	0,25	0,25	0,25	0,24	0,23	0,22	0,21	0,01
Consumption	0,10	0,17	0,22	0,25	0,27	0,27	0,28	0,28	0,27	0,27	0,07
Investment	-0,12	0,25	0,22	0,18	0,16	0,14	0,12	0,11	0,11	0,10	0,01
Scrapping age	-0,03	-0,04	-0,04	-0,04	-0,04	-0,05	-0,05	-0,05	-0,04	-0,04	0,00
Employment	-0,02	-0,03	-0,03	-0,03	-0,02	-0,02	-0,02	-0,02	-0,02	-0,02	0,01
Real wages	0,05	0,06	0,05	0,06	0,06	0,06	0,06	0,06	0,06	0,06	0,00
Production price	0,02	0,10	0,18	0,25	0,33	0,41	0,48	0,55	0,61	0,68	
Nominal exchange rate	0,20	0,27	0,34	0,41	0,48	0,54	0,61	0,67	0,72	0,78	
Real exchange rate	0,12	0,10	0,08	0,07	0,05	0,04	0,03	0,03	0,02	0,01	-0,16
Short-term interest rate*	0,07	0,07	0,07	0,07	0,07	0,07	0,06	0,06	0,06	0,05	
Real interest rate*	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01	0,00
Public balance to GDP*	-0,04	-0,04	-0,03	-0,03	-0,02	-0,02	-0,02	-0,02	-0,02	-0,02	0,00
Trade balance*(Bill. of US$	-16,5	2,6	4,1	3,2	1,7	-0,2	-2,3	-4,5	-6,7	-9,1	3,2
United Kingdom											
Private Production	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,01	0,02
Consumption	0,02	0,02	0,03	0,04	0,04	0,04	0,04	0,04	0,05	0,05	0,03
Investment	-0,05	-0,01	0,01	0,02	0,02	0,03	0,02	0,02	0,02	0,01	0,02
Scrapping age	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Employment	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,02
Real wages	0,00	-0,01	-0,01	-0,01	-0,01	-0,01	0,00	0,00	0,00	0,00	0,00
Production price	0,00	0,00	0,00	0,00	0,00	0,01	0,01	0,01	0,01	0,01	
Nominal exchange rate	-0,16	-0,15	-0,13	-0,12	-0,10	-0,09	-0,08	-0,07	-0,06	-0,06	
Real exchange rate	-0,18	-0,17	-0,17	-0,16	-0,15	-0,14	-0,13	-0,12	-0,12	-0,11	-0,11
Short-term interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	
Real interest rate*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Public balance to GDP*	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Trade balance*(Bill. of US$	39,0	25,0	15,2	7,7	2,5	-1,1	-3,3	-4,6	-5,3	-5,8	7,7

Note: % deviation from baseline, except * (absolute differences from base).

FINAL APPENDIX 1

ECONOMETRIC METHODOLOGY:

Estimation of an equation over a panel countries

Many behavioural equations of Marmotte were estimated over a panel of industrialised countries. Estimating a macroeconomic equation over a panel of countries has become popular. According to this approach the values of the parameters of this equation, but not its specification, may differ or not between countries. Using panel estimation helps to get more robust and precise empirical findings: as these countries share some common structural features, each country estimation benefits from information brought by its partners. Moreover, panel estimation allows to identify deep structural differences between countries.

As the errors terms of the various countries are probably correlated, and as the structure of these correlations are probably more complex than the one allowed by error components models, SUR methods appear the most natural way to make this estimation. However, the presence of endogenous and anticipated explanatory variables requires the use of instrumental variables and GMM, instead of generalised least squares. In both cases, the covariance matrix of the shocks hitting the countries at a same time has a large dimension and is estimated on a rather short time period. This problem is made more complex if we allow for some autocorrelation between shocks. To improve the precision of the estimation of the covariance and the autocovariances of the shocks and to solve the problems which might result from their singularity, we assume that the structure of the shocks can be represented by a limited number of common factors and we use recent developments of factor analysis.

We also develop a strategy of tests, proceeding from general to specific to determine which parameters change between countries and which parameters take the same value among all countries.

In a first paragraph, we present the equation we want to estimate on a panel of countries. The second paragraph gives the GMM estimation method we propose. The third paragraph develops an application of factor analysis to improve the estimation of the covariance matrix of national shocks in the case where these shocks are time independent. The fourth paragraph extends this method to a better estimation of the covariance and the autocovariance of national shocks when they are time dependent. The fifth paragraph gives a strategy of tests to determine which parameters change between countries and which parameters take a common value. It ends by a test of the time dependency of national shocks.

I THE PROBLEM

We want to estimate on a panel of I countries, indexed by i, and on a period of T years, indexed by t, the following system of I equations:

(1) $$y_{it} = g(y_{i,t-1}, x^{a}_{1i,t+1}, x_{2it}, x_{3it}; \boldsymbol{a}_i) + \boldsymbol{e}_{it}; i = 1,..,I; t = 2,..,T-1 .$$

I and T are of the same order of magnitude, and not very high. The y_{it} are the explained variables, the x_{jit} are the explanatory variables, g is a function representing the behavior associated to country i, the a_i are the parameters of this function (they may differ or not between countries). e_{it} is the error term of null expected value[67].

We assume that the error terms of a common year are correlated, and call Ω, of typical element w_{ij}, their covariance matrix. This assumption is consistent with an interpretation of error terms as correlated random shocks affecting the different domestic economies. We will put some structure on these correlations to increase the number of degrees of freedom of our estimation. However, the structure usually proposed by error component models is too restrictive for our needs. We will assume either that the error terms are non autocorrelated or that they are autocorrelated.

If all the explanatory variables were predetermined, that is if the e_{it} were independent of the contemporaneous and past values of the explanatory variables, system (1) could be easily estimated by generalised nonlinear least squares. However, we prefer making more general assumptions. Thus, we assume that variable x_{2it} is predetermined, but that this property is not shared by variable x_{3it}. Moreover, variable $x^a_{1i,t+1}$ represents the forecast at time t of variable x_{1i} for time $t+1$. As this variable is not observed, we follow a suggestion by Wickens (1981), and substitute it by its observed value at time $t+1$: $x_{1i,t+1}$. Thus, we introduce a supplementary error in the equation, which bears on the foreseen value of an explanatory variable for a future time[68]. We have then to estimate a model with errors on variables. The endogeneity of some variables and the error in some others make least squares estimators non consistent.

To overcome these difficulties we allocate to each national equation a vector line of n instrumental variables W_{it} and we assume that the processes they follow are uncorrelated with the processes of the e_{it}.. Then, we will use a two steps GMM method.

[67] Our problem is different from the one of Pesaran and Smith (1995). We directly estimate the various values that a given parameter can take in the various countries without supplementary assumptions. For Pesaran and Smith, the differences between these values are random, and they estimate the expected value and the variance of each coefficient over the set of all countries. They could also compute in their analytic frame the optimal forecast of the values that the coefficients take in the various countries. Thus, if we use the terminology of the econometrics of panels, our approach is similar to that of the models with fixed-effects, and the approach of Pesaran and Smith is similar to the one of component errors models. This last approach gives more precise estimations, if the stronger assumptions it requires are valid.

[68] And then which follows a difference of martingale, independent of the explanatory and explained variables at and before current time.

II ESTIMATION OF THE SYSTEM OF EQUATIONS (1) BY GMM

Let W_i be the matrix of observations for the instruments related to country i, of size $(T-2, n)$. W_{it} is its typical line. Then, we define by $V_t = (\boldsymbol{e}_{1t} W_{1t} \ldots.. \boldsymbol{e}_{It} W_{It})$ the line vector of size In, and by V the matrix with typical line V_t and dimension $(T-2, In)$. The moment's condition is:

(2) $EV_t = 0$

We approximate the theoretical moments by the empirical moments and we get:

(3) $V'\boldsymbol{i} = 0$

where $\boldsymbol{i}$ is a column vector of 1 with dimension $T-2$. Condition (3) cannot be exactly checked in most cases where the total number of instruments is larger than the number of parameters to estimate. Thus, we try to minimise the distance between $V'\boldsymbol{i}$ and 0, by using a distance matrix A, of dimension: (In, In), which is symmetric and positive definite. Thus, we minimise relatively to parameters the expression:

(4) $\boldsymbol{i}'VAV'i$

The efficient choice of matrix A is: $A = \Phi^{-1}$, where Φ. is the spectral density at frequency 0 of $V'\boldsymbol{i}/(T-2)^{1/2}$. We can separate two cases. When the process of the error term is non autocorrelated, the estimation of Φ has for typical element: $\boldsymbol{w}_{ij} W_i' W_j /(T-2)$. Then, to compute A, we must invert this matrix, of dimension (In, In)[69].

When the process of the error term is autocorrelated we note $\Gamma(h) = E\boldsymbol{e}_t \boldsymbol{e}_{t+h}', \forall h \in Z$, its autocovariance function, with typical element $g_{ij}(h)$. Let us choose a kernel $\boldsymbol{k}(\ )$ and a

[69] We have assumed that the covariance matrix Ω, is independent of time (time homoscedasticity), In the computation of Φ we could expect to get estimators of the parameters and of their covariance matrix robust to time-heteroscedasticity by substituting in the expression of Φ, $\hat{\omega}_{ij} W_i' W_j /(T-2)$ by $\sum_{t=2}^{T-1} W_{it}' \hat{e}_{it} \hat{e}_{jt} W_{jt} /(T-2)$. The problem is that this new estimator is the sum of $T-2$ matrices of dimensions (In, In), but of rank 1. Indeed the term of time t is the product of the column vector V_t' by its transpose. Consequently, the rank of the estimator of Φ is at most equal to $T-2$. In most applications it will be less than In, and matrix Φ will be singular so non-invertible. Thus, it seems impossible to build estimators robust to heteroscedasticity for our problem. For the same reason we will assume the autocovariance matrices of the error terms to be independent of time.

bandwith parameter $\boldsymbol{x}$. . Then, the estimation of Φ: has for typical element: $[\boldsymbol{k}(0)\boldsymbol{w}_{ij}W_i^{'}W_j + \sum_{h=1}^{T-3}\boldsymbol{k}(h/\boldsymbol{x}_{T-2})\boldsymbol{g}_{ij}(h)\sum_{t=2}^{T-1-h}W_{it}^{'}W_{j,t+h}]/(T-2)$.

Den Haan and Levin (1996) is a good guide for the choice of the kernel and the bandwith.

In practice we proceed through two steps. In the first step, we assume the errors terms to be non autocorrelated and with a covariance matrix Ω proportional to the identity matrix. Thus, A is the block diagonal matrix, with typical block: $(W_i^{'}W_i)^{-1}$. We minimise criteria (4), and thus we get a first value for the parameters and the residuals. Then, we can compute estimators of the covariance and the autocovariance matrices of the error terms and, in the second step, apply the previous formulae. This second step may be iterated several times.

The covariance matrix of the estimated parameters time $(T-2)^{1/2}$, is asymptotically equal to $(\Delta'\Phi^{-1}\Delta)^{-1}$, where Δ is the matrix of the partial derivatives of $V'i/(T-2)$ relatively to the parameters.

A difficulty is that the estimation of the covariance and autocovariance matrices of the error terms is very imprecise: $\hat{\boldsymbol{e}}_{it}$ is observed for $t = 2,..,T-1$, which makes $T-2$ observations. Yet I is of the order of $T-2$. Thus these matrices are almost singular, or even singular if the number of observed years is smaller than the number of countries. We use factor analysis to put some structure in this matrix; that is some interdependence between the shocks hitting the countries in a way that should appear natural to economists.

III ESTIMATION OF THE COVARIANCE MATRIX OF THE ERROR TERMS WHEN THEY ARE NON AUTOCORRELATED[70]

$\boldsymbol{e}_t$ denotes the vector of error terms for the set of all nations (of dimension I) and for $t = 2,..,T-1$. We denote in the same way the random vector, its realisation and its estimation. We make the following assumptions:

(5) $\boldsymbol{e}_t = \Lambda F_t + u_t$,

F_t represents a column vector of dimension f ; its elements are called common factors. u_t is a column vector of dimension I ; its elements are called specific components. Both are random. Λ is a matrix of dimension (I, f) and is certain. Its elements are called loadings.

$EF_t = Eu_t = 0$, $E(u_t u_t^{'}) = D = diag(d_1,..,d_I)$[71], $E(F_t u_t^{'}) = 0$, $\forall t, \boldsymbol{t}$.

[70] Doz (1998, page 85-161) gives a clear and rigorous introduction to factor analysis in the case of non autocorrelation, and we base on it here. Doz borrows much from Lawley and Maxwell (1971) and from Bartholomew (1987).

[71] $diag$ means a diagonal matrix with the following diagonal elements.

$E(F_t F_\tau') = E(u_t u_\tau') = 0, \forall \tau, t \neq \tau$, $E(F_t F_t') = U_f$ [72].

Then, we deduce:

(6) $\quad \Omega = \Lambda\Lambda' + D$

Instead of having to estimate the $I(I+1)/2$ parameters of Ω, we just have to estimate the $(f+1)I$ parameters of Λ and D (actually the improvement is meaningful only when the number of factors is much smaller than half the number of countries). It can be shown that the maximum likelihood estimators of Λ and D, denoted by $\hat{\Lambda}$ and $\hat{D}$, under the assumption of normality of e_t, are given by conditions:

$\hat{\Omega} = \sum_{t=2}^{T-1} (e_t - m)(e_t - m)'/(T-2)$, where m is the arithmetic mean vector of the e_t over the estimation period.

$1+g_1,..,1+g_I$, are the real positive eigenvalues of $\hat{\Omega}\hat{D}^{-1}$, which are assumed to be different and ranked by decreasing values (actually, the f first g_i must be positive for the computation to be possible),

Γ is the diagonal matrix of dimension (f, f) with diagonal elements: $g_1, . g_f$.

The f columns of $\hat{\Lambda}$ are the f first eigen vectors of $\hat{\Omega}\hat{D}^{-1}$ (related to the f largest eigenvalues) which are normed to check for the identification condition: $\Gamma = \hat{\Lambda}\hat{D}^{-1}\hat{\Lambda}$.

The estimation procedure is iterative. First, we give an initial value to $\hat{D}$: D_0. Then we compute the eigenvalues and the eigen vectors of $\hat{\Omega}D_0^{-1}$,and consequently Λ_0. Then, we compute D_1 which is the diagonal matrix, the diagonal elements of which are the same as for $\hat{\Omega} - \Lambda_0\Lambda_0'$, and we start again. This procedure appears to converge easily in applications, although to our knowledge there do not exist mathematical results proving this property. More sophisticated estimation methods exist and are given by Doz[73].

The choice of the initial value D_0 is a supplementary problem. We denote by R_i^2 the square of the multiple correlation coefficient between the *ith* component of e_t and the

---

[72] U_f represents an identity matrix, which in this case is of dimension (f, f).

[73] The empirical covariance $\hat{\Omega}$ and its estimated approximation $\hat{\Lambda}\hat{\Lambda}' + \hat{D}$ have the same diagonal. This results from the fact that the factor representation does not change variances, but simplifies the structure of the covariances by assuming that it results from a small number of common factors.

$I-1$ other components, and by $\hat{w}_{ij}$ the typical element of matrix $\hat{\Omega}$. Then, we choose: $d_{i0} = \hat{w}_{ii}(1-R_i^2)$.

Another difficulty is the choice of the number of factors f. A simple method is to compute a matrix of the same dimension as $\hat{\Omega}$, the non diagonal terms of which represent the correlations between the components of vector e_t, and the diagonal terms of which are the R_i^2. Then we make a principal component analysis of this matrix, and we keep as many factors as there exists non-negligible positive eigenvalues.

This *a priori* test is sufficient at the beginning of a succession of iterations of GMM, when the fact that matrix Φ may be a little wrong bears no serious consequences. However, an *a posteriori* test of the validity of the choice of the number of factors, more rigorous, must be made at the last step of GMM. This test, of the likelihood ratio kind, uses as null hypothesis that the number of factors is equal to f. The alternative hypothesis is that there does not exist any constraint on the covariance matrix Ω. The statistics of the test is:

(7) $x = -(T-2)\sum_{j=p+1}^{I} \ln(1+g_j)$.

This statistics asymptotically verifies a c^2 with a number of degrees of freedom equal to $[(I-p)^2-(I+p)]/2$. Bartlett suggests substituting, in the expression of x, the number of observations: $T-2$, by: $T-2-(2I+5)-2p/3$, when the number of observations is low, which is the situation we face here.

IV ESTIMATION OF THE COVARIANCE AND AUTOCOVARIANCE MATRICES OF THE ERROR TERMS WHEN THEY ARE AUTOCORRELATED

We assume now that each common factor and each specific component follows a weakly stationary process, and may present autocorrelation. We do not make any normality assumption. All the other assumptions of the previous section are kept unchanged. In particular, the factors and the specific components are non correlated to one another. Under these assumptions, the estimator of the last section is a M-estimator. Doz and Lenglart (1999) show that it is consistent. This estimator is non efficient, and does not give any information on the stochastic process followed by the factors and the specific components. However, the computation of this estimator allows the computation of a pseudo-score test of the number of common factors, developed by Doz and Lenglart[74]. Let us make, as in previous section, the null hypothesis that the covariance of the error terms can be represented by a model with f factors. The alternative hypothesis is that there does not

[74] Doz (1998) shows that when e_t is non autocorrelated, the likelihood ratio test of the previous section is asymptotically equivalent to the pseudo-score test that we are going to present.

exist any constraint on the covariance of process e_t. We first have to introduce some new notations. We will represent by index 0 the true value of a parameter and by a ^ its M-estimated value. We call:

$$q = \begin{pmatrix} vec\Lambda \\ d \end{pmatrix}, \text{with: } d = (d_1,..d_I).$$

$h(q)$ is the application which associates to q the vector: $vech(\Lambda\Lambda' + D)$.

E_I is the duplication matrix of order I with dimension $(I^2, I(I+1)/2)$, which verifies for all symmetric matrix M of dimension I : $vecM = E_I vechM$.

$E_I^+ = (E_I' E_I)^{-1} E_I'$ is the pseudo-inverse of E. More generally, exponent – means pseudo-inverse and exponent + means generalised inverse.

We will assume now that e_t follows a Gaussian stationary process, and we note: $\forall h \in Z$,

$$\Gamma(h) = E(e_t e_{t+h}'),\ B_0 = \sum_{h \in Z} \Gamma(h) \otimes \Gamma(h).$$

$$J_0 = D_I' \Gamma(0)^{-1} \otimes \Gamma(0)^{-1} D_I / 2$$

$$P = \frac{\partial h}{\partial q'}(q_0) \left(\frac{\partial h'}{\partial q}(q_0) J_0 \frac{\partial h}{\partial q}(q_0) \right)^+ \frac{\partial h'}{\partial q}(q_0) J_0,\ M = U_{I(I+1)/2} - P.$$

Then, the statistics of the pseudo-score test is:

$$x = [(T-2)/2] vech(\Lambda\Lambda' + D - \hat{\Omega})' (\hat{M} E_I^+ B_0 E_I^{+'} \hat{M}')^- vech(\Lambda\Lambda' + D - \Omega).$$

This statistics asymptotically verifies a c^2 with a number of degrees of freedom equal to $[(I-p)^2 - (I+p)]/2$.

Practically, B_0 is computed as a sum including the $\Gamma(h)$ which are different enough from 0. We can also check that increasing the range of the sum has no significant effect on the numerical value of the test. When we have determined the number of factors, we can estimate the model of the error terms by computing the likelihood function with a Kalman filter as Stock and Watson (1993) suggested.. Let us assume to simplify the presentation that each common factor follows an ARMA(1,1) and that each specific component follows an AR(1).

The model can be written:

$$(8)\ e_t = \Lambda F_t + u_t$$

$$F_t = jF_{t-1} + n_t - qn_{t-1}$$

$$u_t = ru_{t-1} + z_t$$

$\boldsymbol{j}$ and $\boldsymbol{r}$ are diagonal matrices of respective dimensions f and I . $\boldsymbol{n}_t$ and $\boldsymbol{z}_t$ are random vectors of dimensions f and I . Their components are non correlated to one another and non autocorrelated. Moreover, we will assume that they are Gaussian. The model can be rewritten as an inobservable component model in a state-measure form:

$$(9)\ \boldsymbol{e}_t = \begin{pmatrix} \Lambda & 0 & U_I \end{pmatrix} \begin{pmatrix} F_t \\ \boldsymbol{n}_t \\ u_t \end{pmatrix}$$

$$\begin{pmatrix} F_t \\ \boldsymbol{n}_t \\ u_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{j} & -\boldsymbol{q} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{r} \end{pmatrix} \begin{pmatrix} F_{t-1} \\ \boldsymbol{n}_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} U_p & 0 \\ U_p & 0 \\ 0 & U_I \end{pmatrix} \begin{pmatrix} \boldsymbol{n}_t \\ \boldsymbol{z}_t \end{pmatrix},$$

or, with an evident change in notations:

$$(10)\ \boldsymbol{e}_t = Z\boldsymbol{a}_t$$

$$\boldsymbol{a}_t = C\boldsymbol{a}_{t-1} + \boldsymbol{h}_t$$

$\boldsymbol{e}_t$ is the measure vector which is observable. $\boldsymbol{a}_t$ is the state vector. $\boldsymbol{h}_t$ is the perturbation vector, which is non autocorrelated, and the components of which are nor correlated to one another. We want to estimate elements of matrices Z and C and the variances of the perturbations. This can be easily done by maximum likelihood, using optimal forecasts .of the state vector and the covariance matrices of the forecast error, which are given by the use of Kalman filter (see Harvey (1989)).

A practical problem is that we must choose the number of lags of the ARMA processes which fit the best the dynamics of the common factors and of the specific components. The only solution we see is an error an trial method, which checks for the significativity of the estimated parameters and the quality of the innovations $\boldsymbol{h}_t$.

When the estimation has been made, it is easy to derive the autocovariance matrices of $\boldsymbol{e}_t$, $\Gamma(h)$, under the assumption that model (8) is valid.

V TESTS

We will first present two tests that are easy to implement: the over identifying restriction test of Hansen and the tests of restrictions on parameters. Then, we will develop a strategy based on these tests to identify if the values parameters change or nor between countries. Finally, we will present a last test which concerns the autocorrelation of the error terms.

The over identifying restriction test of Hansen. If the model and the instruments are valid, the objective function of the second step of the GMM $(i'\hat{V}\Phi^{-1}\hat{V}'i)/(T-2)$, where the hat identifies estimated values, follows a c^2 with s degrees of freedom[75].

Tests of restrictions on parameters. A ^ identifies the results of the estimation of the non-constrained model. A ˜ identifies the results of the estimation of the model constrained by r equalities between parameters. Then, the statistics of the likelihood ratio $LR = (i'\tilde{V}\tilde{\Phi}^{-1}\tilde{V}'i - i'\hat{V}\tilde{\Phi}^{-1}\hat{V}'i)/(T-2)$ follows a c^2 with r degrees of freedom[76]. This expression uses twice the estimation of Φ got at the last step of the GMM estimation of the constrained model. Then, the estimation of the non-constrained model is very simple and is made in one step[77].

a) *A strategy of tests of equality of parameters between countries.* To test for the equality or of the difference of the values of each parameter between countries, we choose to progress from general to specific. In a first series of null hypotheses, we assume that all the coefficients of the model differ between countries, except one. Then we continue with two coefficients common to all countries, etc. until the tests induces us to stop.

We give the example of our strategy in the case of an equation including three parameters: a_i, b_i and c_i. We test hypotheses relative to the equality of the values taken by one of these parameters over the total of all countries (which can be denoted, for instance: $c_i = c_j$), against the alternative that this parameters takes values which differ among countries ($c_i \neq c_j$). To retain the null hypothesis, we require that the likelihood ratio test again the alternative hypothesis, and the Hansen tests under the null hypothesis, have both p-values larger than 5%. Our strategy of nested tests is given in the following diagrams. At each step, a null hypothesis is accepted if it is not rejected against any of the associated alternative hypothesis. If, for a given alternative hypothesis, one of the associated null hypotheses is not rejected, this alternative hypothesis is rejected. This second criteria can be criticised. Indeed, an alternative hypothesis H1 can fail to reject the null hypothesis H0, but H0 is rejected against another alternative hypothesis H1'. By rejecting H1, we implicitly assume that this hypothesis includes wrong features, which do not appear in H1'. At the end of the series of nested tests, it may be possible to retain several configurations of equalities and differences of parameters between countries. Then, we could try to choose

[75] s is the difference between the numbers of instruments and parameters (see Appendix 1).

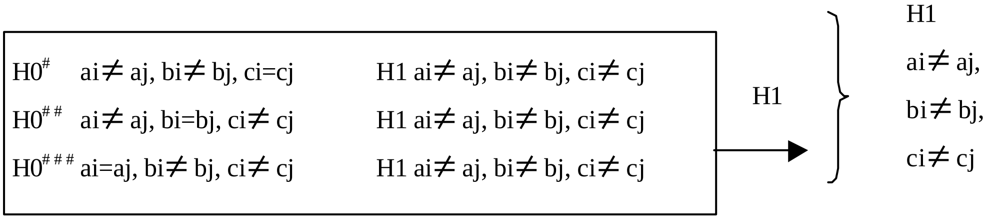
[76] Another solution would have been to use a Lagrange test: $\mathrm{LM} = \iota' V\Phi^{-1}\Delta(\Delta'\Phi^{-1}\Delta)^{-1}\Delta'\Phi^{-1}V'\iota/(T-2)$, where Δ is the matrix of the partial derivatives of $V'\iota/(T-2)$ relatively to the parameters of the unconstrained system of equations. Under the null hypothesis, LM follows a c^2 with r degrees of freedom, where r is the number of constraints tested on parameters. However, a test of the likelihood ratio is a little easier to implement.

[77] In the case where the non-constrained model is just identified, the statistics of this test is numerically equal to the statistics of the over identification test of Hansen (for a common value of Φ of course).

between them by using non-nested tests of the J kind (see Davidson and MacKinnon (1993), chapter 11), or by economic arguments.

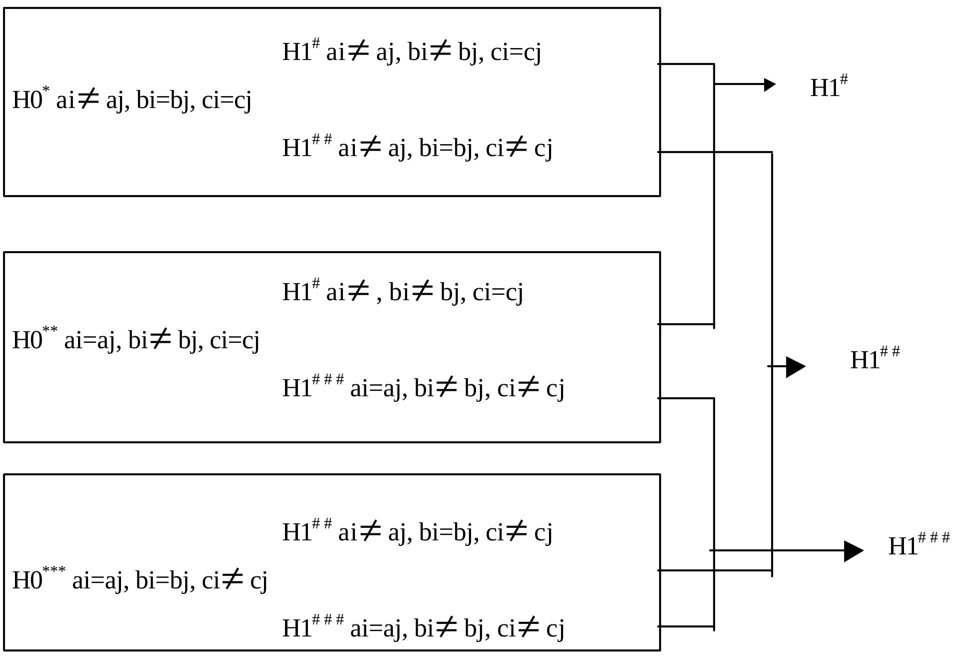
First step

The alternative hypothesis H1 is that the three parameters differ between countries. The three null hypotheses H0#, H0## and H0### are that one of these parameters takes the same value across countries. If the tests of these null hypotheses reject the three null hypotheses, we retain the alternative assumption. Otherwise, we go to the second step.



Second step

In the following diagram we assume that each of the three null hypotheses of step 1 has not been rejected, and now represents as many alternative hypotheses denoted by H1#, H1## et H1###. (Otherwise, some of these alternative hypotheses should not appear in the diagram.) Each alternative hypothesis assumes that two parameters take different values across countries and is associated with two null hypotheses where only one of these parameters changes across countries. The total number of possible null hypotheses is equal to three: H0*, H0** and H0***. Each of them is associated with two alternative hypotheses.



Third step

The diagram corresponds to the case where the three null hypotheses of the second step have not been rejected. Then, they become as many alternative assumptions denoted H1*, H1** and H1***. Now, we have only one null hypothesis, H0 where the three parameters take common values across countries. If H0 is rejected against one or several alternative hypotheses, we retain this (these) last hypothesis. Otherwise, we retain the null hypothesis H0.

	$H1^{*}$ ai $\neq$ aj, bi=bj, ci=cj	$\rightarrow$ H1*
H0 ai=aj, bi=bj, ci=cj	$H1^{**}$ ai=aj, bi $\neq$ bj, ci=cj	$\rightarrow$ H1**
	$H1^{***}$ ai=aj, bi=bj, ci $\neq$ cj	$\rightarrow$ H1***

This series of nested tests is used to establish if the values of the various parameters change or not with countries. To do that in a rigorous way we must keep the same instruments for all the tests.

d) *The autocorrelation of error terms*

The application of factor analysis to the estimation of the covariance of shocks is simpler to implement when we assume that the error terms of the equation are non autocorrelated. However, if this assumption is wrong, and if we include in the instruments lagged endogenous variables, the moment condition (1) may become non verified. Moreover, our choice of the weights matrix $A = \Phi^{-1}$, is no more efficient, and the covariance matrix of the estimated parameters, as the Hansen statistics, are no more computed adequately. Let us assume that we have estimated the model using as instruments, endogenous variables with a lag of m years. We have to evaluate if m makes a long enough lag. To do that we reestimate the equations with the endogenous variables lagged m+1 years as instruments. We compute the Hansen statistics and its *p-value*. Then, we add to the lists of instruments the endogenous variables with a lag of m years. We still compute the Hansen statistics and its *p-value*. Then, we compute the difference between both Hansen statistics. If the new instruments are valid this difference follows a c^2 with a number of degrees of freedom equal to the number of supplementary variables time the number of countries. We compute its *p-value*. If the three *p-values* are high enough we conclude that using the endogenous variables with a lag of m years as instruments is justified.

FINAL APPENDIX 2
TROLL EQUATIONS

I LIST OF VARIABLES (46 VARIABLES)

ENDOGENOUS VARIABLES

A = Age of the oldest production units in working order at the beginning of the current period

ACT= Real domestic economic activity

B1 = Net liabilities of general government at the beginning of the year

C = Private final consumption expenditures (total households) at 1990 prices

CU = Private final consumption expenditures (unconstrained households) at 1990 prices

CC = Private final consumption expenditures (constrained households) at 1990 prices

EP = Total industries employment

EPI = Number of production units implemented during the current period and employment required by these units

ET = Total employment

EXM = Exports of non primary goods and services at 1990 prices

EXMA = Exports of non primary goods and services at 1990 prices adjusted for the world discrepancy

FACTM = Foreign activity (non primary goods and services)

GDP = Gross Domestic Product at 1990 prices

IMM = Imports of non primary goods and services at 1990 prices – national currency

INFL = Inflation rate of the GDP deflator at factor cost at the current period

INFLE = Inflation rate of the GDP deflator at factor cost expected for the next period

INFLC = Inflation rate of consumer price at the current period

IS = Short term nominal interest rate, percent (excluding the US one)

J = Gross fixed capital formation at 1990 prices by total Industries plus increase in stocks, including adjustment costs

JJ = Investment of the period

KAPA = reduced capital intensity of the new production unit (capital divided by efficiency and employment) built at the current period and productive from the next period

NFA1 = Net foreign assets in current US dollar at the beginning of the year

NXC = Net exports of primary commodities at 1990 prices

NXCA = Net export of commodities adjusted for the world discrepancy

OTR = Other taxes rate

P = GDP deflator at factor cost

PA = Absorption deflator (excluding consumption)

PBAL = Primary surplus of Government

PC = Private consumption deflator

PCO = World price of primary commodities in US dollar

PFM = Price of competitors on export market in domestic currency (non primary goods and services)

PMM = Import price for non primary goods and services in domestic currency

PMMA = Import price for non primary goods and services in domestic currency adjusted for the world discrepancy

PMP = GDP deflator at market prices

PXM = Export prices for non primary goods and services in domestic

T = Lifetime of the unit implemented during the current period

TB = Trade balance in domestic currency

TT = Government income

VAT = Value added tax receipts (plus import duties and other adjustments)

WPRR = Real wage cost rate, private sector

WPRRA = Real wage in efficiency unit, private sector

WTCR = World trade discrepancy for primary commodities in constant dollar

WTM = World trade discrepancy in dollar nominal terms (non primary goods and services)

WTMR = World trade discrepancy in dollar real terms (non primary goods and services)

YQ = Production of total industries – imputed bank service charges

YQI = Production capacity of a production unit built in the current period

EXOGENOUS VARIABLES

AY = Age of the oldest production units in working order at the beginning of the current period (integer part)

BEN = Real unemployment benefits per person unemployed

BILE = Share of long term external debt (long term loans + shares)

BILG = Share of long term public debt (long term loans + long term bonds)

CTBR = Current transfer balance, domestic currency, in constant price

E = Nominal exchange rate against dollar; E90 is the nominal exchange rate in 1990

E_RW = Nominal exchange rate against dollar of the rest of the world; E90_RW is this rate in 1990

EG = Employment, producers of government services

G = Government final consumption expenditures in private goods and services at 1990 prices plus private final consumption expenditures of private non-profit institutions serving households, excluding wages

IL = Long term nominal interest rate (percent)

JG = Gross fixed capital formation at 1990 prices by producers of government services and private non-profit institutions serving households

LF = Labour force employment total

ONDGR = Other net receipt of General Government (in constant price)

OTR_EXO = Exogenous rate of other taxes set by government.

PT = Technical progress (equal to P*WPRR in 1990 and grows at the exogenous rate of technical progress: SS_GGDP/SS_POP)

PXM_RW = Rest of the World export price for non primary goods in current dollar.

PMM_RW = Rest of the World import price for non primary goods in current dollar.

RPC = Relative price correction relative to the difference between PA and PC not taken into account by value added tax evolution. We have: RPC=(PA/PC)*(1+VATR)-1, where the variables of the right-hand side are fixed at their historical or reference account values. At the base year: RPC=VATR

SCCR = Apparent rate of social security contributions paid by households and private unincorporated enterprises,

SDQ = Statistical discrepancy in constant price on the goods and services market equilibrium in volume

SDVR = Statistical discrepancy in constant price on the goods and services market equilibrium in value

SC_i = Share of the generic country in the total imports of commodities of country i (SCk_i =Share of country k in the imports of country i; SCi = Share of country i in the imports of the generic country). The exporting countries are the 17 industrialised countries, the importing countries include also the rest of the world.

SM_i = Share of the generic country in the imports of non primary goods and services of country i (SMk_i =Share of country k in the imports of country i ; SMi = Share of country i in the imports of the generic country) The exporting countries are the 17 industrialised countries, the importing countries include also the rest of the world

TCR = Corporate tax rate

TLR = Rate of direct taxes paid by households; based on labour income less social security contribution

TRR= Net general government transfers in constant price

TY = Integer part of the lifetime of the unit implemented during the current period

VADR = Value added discrepancy in constant price

VATR = Value added tax rate (VATR90 = Value added tax rate in 1990)

WGRR = Real wage cost rate, public sector

WM_i = Share of the generic country's exports to country i in the generic country's total export

WTSC = Share of the generic country in the 17 industrialised countries net exports of primary commodities

WTSM = Share of the generic country's exports in world exports of non primary goods and services in 90 dollar

XF = reduced firing cost (i.e. deflated by P.PT)

Total compensation in our model overstate the National Account value because here, all the workers (including self-employed worker) are suppose to receive the private sector wage rate.

II EQUATIONS OF THE DYNAMIC MODEL[78]

II.1 National accounting identities (13 equations)

`(1) Goods and services market equilibrium, value INDEX(1990=1)`

```
PMP= (PC*C+PA*(G+J+JG)+PC*WGRR*EG+PXM*EXMA-PMMA*IMM+
PCO_WO*(E/E90)*NXCA +PMP*SDVR)/GDP+PMP*RES_PMP
```

`(2) Value Added Tax (in millions of domestic currency at current prices)`

```
VAT= C*PC*VATR/(VATR+1)+PC*RES_VAT
```

`(3) GDP deflator at factor costs INDEX(1990=1)`

```
log(((GDP-RES_GDP)*PMP-EG*PC*WGRR-VAT-PMP*VADR)/(YQ*P))
=1/lambda*log(P/(P(-1)*(SS_INFL)))
```

`(4) Goods and services market equilibrium, volume (in millions of domestic currency at constant prices)`

```
GDP = C+G+(J-RES_J)+JG+PC*WGRR*EG/PMP + EXMA- IMM+NXCA+SDQ
```

[78] Some variables are expressed in millions of domestic currency. But for Spain, Japan and Italy, they are actually expressed in billions.

(5) Consumer price INDEX(1990=1)

```
LOG(PC)=pc1*LOG(P)+(1-pc1)*pc2*LOG(PMMA)+(1-pc1)*(1-pc2)
*LOG(E/E90*PCO_WO)+LOG((1+VATR)/(1+VATR90))+LOG(RES_PC)
```

(6) Absorption price INDEX(1990=1)

```
PA=PC*(1+RPC)/(1+VATR)+PA*RES_PA
```

(7) Receipts of general government(in millions of domestic currency at current prices)

```
TT=VAT+(SCCR)*(P*WPRR*EP+PC*WGRR*EG)+TLR*(1-SCCR)*(P*WPRR*EP+PC
*WGRR*EG)+TCR*(P*YQ-(P*WPRR*EP))+OTR*PMP*GDP+PMP*RES_TT
```

(8) Other taxes

```
OTR=otr0*OTR_EXO+(1-otr0)*(otr1*B1/(PMP*GDP)+otr2)+RES_OTR
```

(9) Public balance (including the debt service)(in millions of domestic currency at current prices)

```
PBAL = TT-(G+JG)*PA-EG*PC*WGRR-(PMP*TRR-BEN*P*(LF-ET))-BEN*P*
(LF-ET)-((1-BILG)*IS/100+BILG*IL/100)*B1+PMP*ONDGR+PMP*RES_PBAL
```

(10) Net liabilities of general government (in millions of domestic currency at current prices)

```
B1 = (1-BILG(-1)+BILG(-1)*(IL(-1)/IL))*B1(-1)-PBAL(-1)+PMP(-
1)*RES_B1
```

(11) Trade balance (in millions of domestic currency at current prices)

```
TB = PXM*EXMA - PMMA*IMM + (E/E90)*PCO_WO*NXCA+PMP*RES_TB
```

(12) Short-term net foreign assets, USD (in millions of current US dollars)

```
NFA1 = ((1-BILE(-1))*(1+IS_US(-1)/100)+BILE(-1)*(IL_US(-1)/100
+IL_US(-1)/IL_US))*NFA1A(-1)+TB(-1)/E(-1)+PMP_US(-1)*CTBR(-1)/
E90(-1)+PMP_US(-1)*RES_NFA1
```

(13) Total employment (in millions)

```
ET = EP + EG + RES_ET
```

II.2 Households' consumption

(14) Unconstrained households' consumption (in millions of domestic currency at constant prices)

```
0=((CU*(1-RES_CU))/L-cu0*CU(-1)/L(-1))**-cu1)-cu2*(cu0+
(1+IS(+1)/100)*1/INFLC(+1))*(CU(+1)/L(+1)-cu0*(CU*(1-RES_CU))
/L)**(cu1)+cu0*(cu2**2)*(1+IS(+1)/100)*1/INFLC(+1)*(CU(+2)
/L(+2)-cu0*CU(+1)/L(+1))**(-cu1)
```

(15) Constrained households' consumption (in millions of domestic currency at constant prices)

```
CC=cc0*((1-TCR)*(P*YQ-P*WPRR*EP)+(1-SCCR)*(1-TLR)*(P*WPRR*EP
+PC*WGRR*EG)+PMP*TRR)/PC+RES_CC
```

(16) Households' consumption (in millions of domestic currency at constant prices)

```
C=CU + CC + RES_C
```

II.3 Monetary policy

(17) Interest rate term structure (in percentage points)

```
IL=100*(1+IS/100)/(1+100/IL(+1))+RES_IL
```

(18) Inflation rate of the GDP deflator expected for the next period (1+inflation rate)

```
INFLE = INFL(+1)+RES_INFL
```

(19) GDP deflator at factor cost INDEX(1990=1)

```
P = INFL*P(-1)+RES_P
```

(20) Inflation of Consumer Prices (1+inflation rate)

```
INFLC = PC/PC(-1)+RES_INFLC
```

(21) Nominal exchange rate ratio (except for US)

```
DEP = E / E(-1) + RES_DEP
```

(22) Net Foreign assets as % of GDP Price (except for US)(in millions of domestic currency at constant prices)

```
NFA11 = (NFA1*E(-1)/(PMP(-1))) + RES_NFA11
```

(23) Net Foreign assets as % of GDP, for US

```
NFA11_US = (NFA1_US/(PMP_US(-1)*GDP_US(-1))) + RES_NFA11_US
```

(24) Nominal short term interest rate US (in percentage points)

```
1+IS_US/100 = SS_INFL_US*(SS_GGDP_US)**cu1_US/cu2_US + is0_US*(INFL_US-SS_INFL_US) - premf0_US*(NFA11_US(+1)/100) + RES_IS_US/100
```

(25) Uncovered interest rate parity (except for US and the euro area) (number of domestic currency unit per US dollar)

```
1+IS/100 - PREMF/100  =(DEP(+1)*(1+IS_US/100)) +RES_E
```

(26) Risk premiun on net foreign assets (except for US and euro area) (in percentage points)

```
PREMF = -premf0*NFA11(+1)/GDP+SS_INFL*RES_PREMF
```

(27) Nominal short term interest rate (except for US and euro area) (in percentage points)

```
1+IS/100 = SS_INFL*(SS_GGDP)**cu1/cu2+ is0*(INFL-SS_INFL) +RES_IS/100
```

EURO ZONE

(28) Risk premiun on net foreign assets (in percentage points)

```
PREMF = -premf0*NFA11(+1)/GDP+SS_INFL*RES_PREMF
```

(29) Euro zone exchange rate parity (except Germany) (number of domestic currency unit per US dollar)

```
E = E_GE * E99/E99_GE + RES_E
```

(30) Uncovered interest rate parity (except Germany) (in percentage points)

```
1+IS/100 - PREMF/100  = DEP(+1)*(1+IS_US/100) +RES_IS
```

(31) Uncovered interest rate parity EURO versus USD (number of domestic currency unit per US dollar)

```
1+IS_GE/100 - PREMF_GE/100 = DEP_GE(+1)*(1+IS_US/100)
+RES_E_GE
```

(32) BCE monetary policy (in percentage points)

```
1+IS_GE/100 = SS_INFL_GE*(SS_GGDP_GE)**cu1_GE/cu2_GE+is0_GE*
(gprod{jj} (INFL_{jj}**W_{jj})-SS_INFL_GE)+RES_IS_GE/100
```

where *jj* includes the euro area countries.

II.4 Foreign trade

(33) Weighted average of foreign activity (in millions of domestic currency at constant prices)

```
FACTM = gsum{i}(SM_{i}*IMM_{i}/E90_{i}*E90)+RES_FACTM
```

where *i* includes the 17 modelled countries and the rest of the world.

(34) Prices of competitors on export markets in domestic currency INDEX(1990=1)

```
LOG(PFM) = gsum{i}((WM_{i}/(1-SM_{i}))*gsum{k}(SM{k}
_{i}*LOG((PXM_{k}*(E/E_{k})/(E90/E90_{k})))))-LOG((PXM))
*gsum{i}(WM_{i}*SM_{i}/(1-SM_{i}))+LOG(RES_PFM)
```

where *i* and *k* include the 17 modelled countries and the rest of the world.

(35) Exports of manufactured goods and services (volume) (in millions of domestic currency at constant prices)

```
LOG(EXM) = LOG(EXM(-1))+exm0+exm1*DEL(1: LOG((FACTM)))+exm2*DEL
(1: LOG((PFM)))-exm2*DEL(1: LOG((PXM)))+exms*(LOG((EXM(-1)))-
LOG((FACTM(-1)))-exm3*Log((PFM(-1)/PXM(-1))))+LOG(RES_EXM)
```

(36)Export prices for manufactured goods in domestic currency INDEX(1990=1)

```
LOG(PXM)= LOG(PXM(-1))+pxm0+pxm1*DEL(1: Log((P)))+pxm2*DEL(1:
LOG((PFM)))+pxms*(log((PXM(-1)))-(1-pxm3)*LOG((P(-1)))-
pxm3*LOG((PFM(-1))))+LOG(RES_PXM)
```

(37) Real domestic economic activity (in millions of domestic currency at constant prices)

```
ACT= act1*C +act2*(J+JG) +act3*G +act4*EXM+ RES_ACT
```

(38) Imports of manufactured goods (volume)(in millions of domestic currency at constant prices)

```
LOG(IMM)= LOG(IMM(-1))+imm0+imm1*Log((ACT/ACT(-1)))+imm2
*Log((PA*PMM(-1)/PA(-1)/PMM))+imms*(Log((IMM(-1)))-Log((ACT(-1)
))-imm4*Log((PA(-1)/PMM(-1))))+LOG(RES_IMM)
```

(39) Manufactured goods import price INDEX(1990=1)

```
LOG(PMM) =gsum{i}(SM{i}*LOG((PXM_{i})/(E_{i}/E90_{i})*(E/E90)
))+LOG(RES_PMM)
```

where *i* includes the 17 modelled countries and the rest of the world.

(40) Net exports of primary commodities (volume) (in millions of domestic currency at constant prices)

```
NXC=nxc1*ACT+RES_NXC
```

(41) World price of primary commodities INDEX(1990=1)

```
LOG(PCO_WO) = LOG(P_US)+LOG(RES_PCO_WO)
```

(42) World trade discrepancy in manufactured goods and services (in millions of constant US dollars)

```
WTMR_WO = (gsum{j}((IMM-EXM)/E90)+(IMM_RW-EXM_RW*P_US/SS_
P_US)/E90_RW)+RES_WTMR_WO
```

where *j* includes the 17 modelled countries.

(43) Export volume - manufactured goods, adjusted for real world trade discrepancy (in millions of domestic currency at constant prices)

```
EXMA = EXM+(WTSM/(1-WTSM_RW))*WTMR_WO*E90+RES_EXMA
```

(44) Import price for manufactured goods, adjusted for nominal world trade discrepancy INDEX(1990=1)

```
PMMA = PMM-(WTSM/(1-WTSM_RW))*(gsum{j}(IMM*PMM/E-EXMA*PXM/E)+
IMM_RW*PMM_RW/E_RW-PXM_RW*EXM_RW*P_US/SS_P_US/E_RW)*E/IMM+
PXM/IMM*RES_PMMA
```

where *j* includes the 17 modelled countries.

(45) World trade discrepancy for commodities (in millions of constant US dollars)

```
WTCR_WO = gsum{i}((NXC_{i})/E90_{i})+ RES_WTCR_WO
```

Where *i* includes the 17 modelled countries and the rest of the world.

(46) Export volume - primary commodities, adjusted for real world trade discrepancy (in millions of domestic currency at constant prices)

```
NXCA = NXC-(WTSC/(1-WTSC_RW))*WTCR_WO*E90+RES_NXCA
```

(47) Net foreign assets adjusted for world discrepancy, USD (in millions of current US dollars)

```
NFA1A = NFA1 -(WTSM/(1-WTSM_RW))*(gsum{i}(NFA1_{i})) +P(-1)*RES_NFA1A,
```

where *i* includes the 17 modelled countries and the rest of the world.

II.5 Supply side blocks

(48) Production function

```
YQI=z*RES_YQI*(alpha*KAPA**(1-1/sigma)+(1-alpha))**(sigma/(sigma-1))
```

(49) Goods and services market equilibrium, volume (in millions of domestic at constant prices)

```
J = ci0*(JJ-RES_JJ)
```

(50) Investment excluding installation costs (in millions of domestic at constant prices)

```
JJ=PT*KAPA*(EPI-RES_EPI)
```

(51) Real wage rate (wage curve)

```
log(WPRR) = wprr0 +(wprrs*log(YQ(-1)/EP(-1))+(1-wprrs)*
log(YQ/EP)) +wprr1*(wprrs*log(WDG(-1))+(1-wprrs)*log(WDG))
+wprr2*(wprrs*log(ET(-1)/LF(-1))+(1-wprrs)*log(ET/LF))+((1-
wprrs)*log(INFLE(-1)/INFL)+wprrs*log(INFLE))+log(RES_WPRR),
```

(52) Real wage rate, in efficiency units

```
WPRR = (WPRRA-RES_WPRRA)*PT
```

(53) Wedge

```
WDG = PC/((1-SCCR)*(1-TLR)*P)+RES_WDG
```

(54) Expected life time of the new production units (in years)

```
0=gsum{Y2}(((1-sign(TY-{Y2}-0.5)*sign(TY-{Y2}+0.5))/2)
*(log(INFLE({Y2})*(1-delta)/(1+(IS({Y2})+PREMK({Y2}))
/100))*(((T-TY)-RES_T)*(1-TCR({Y2}+1))*((SS_GGDP
/SS_POP)**(-TY-(T-TY)+RES_T)*YQI-(WPRRA({Y2})**(1-(T-
TY)+RES_T)*WPRRA({Y2}+1)**((T-TY)-RES_T)))-(XF({Y2})**
(1-(T-TY)+RES_T)*XF({Y2}+1)**((T-TY)-RES_T)) )+(1-TCR
({Y2}+1))*((SS_GGDP/SS_POP)**(-TY-(T-TY)+RES_T)*YQI-
(WPRRA({Y2})**(1-(T-TY)+RES_T)*WPRRA({Y2}+1)**((T-TY)
-RES_T)))-((T-TY)-RES_T)*(1-TCR({Y2}+1))*log(WPRRA
({Y2}+1)/WPRRA({Y2}))*(WPRRA({Y2})**(1-(T-TY)+RES_T)
*WPRRA({Y2}+1)**((T-TY)-RES_T))-log(XF({Y2}+1)/XF
({Y2}))*(XF({Y2})**(1-(T-TY)+RES_T)*XF({Y2}+1)**((T-TY)-
RES_T))-log(SS_GGDP/SS_POP)*(((T-TY)-RES_T)*(1-TCR
({Y2}+1))*(WPRRA({Y2})**(1-(T-TY)+RES_T)*WPRRA({Y2}+1)**
((T-TY)-RES_T))+(XF({Y2})**(1-(T-TY)+RES_T)*XF({Y2}+1)**
((T-TY)-RES_T)))))
```

(55) Capital intensity of the new production units

```
(z**(1-1/sigma))*alpha*((YQI/(KAPA-RES_KAPA))**(1/sigma))
*(gsum{Y3}(((1+sign(TY-{Y3}+0.5))/2)*(1-TCR({Y3}))*(1-
delta)**({Y3}-1)*(product(i=1 to {Y3}: INFLE(i-1)/(1+(IS(i-
1)+PREMK(i-1))/100))))+gsum{Y3}(((1-sign(TY-{Y3}-0.5)
*sign(TY-{Y3}+0.5))/2)*(T-TY)*(1-TCR({Y3}+1))*((INFLE({Y3})
*(1-delta)/(1+(IS({Y3})+PREMK({Y3}))/100))**((T-TY)))*
(1-delta)**({Y3}-1)*(product(i=1 to {Y3}: INFLE(i-1)
/(1+(IS(i-1)+PREMK(i-1))/100)))))= ci0/(1-delta)
```

<u>(56) Factor cost frontier</u> (1+expected inflation rate)

```
gsum{Y3}( ((1+sign(TY-{Y3}+0.5))/2) * (1-TCR({Y3}))*(1-
delta)**({Y3}-1) * ( z**(1-1/sigma)*(1-alpha)*YQI**(1/sigma)-
((SS_GGDP/SS_POP)**{Y3})*WPRRA({Y3}) )*(product(i=1 to {Y3}:
INFLE(i-1)/(1+(IS(i-1)+PREMK(i-1))/100))))+ gsum{Y3}
( ((1-sign(TY-{Y3}-0.5)*sign(TY-{Y3}+0.5))/2)*((INFLE({Y3})
*(1-delta)/(1+(IS({Y3})+PREMK({Y3}))/100))**((T-TY)))
*(1-delta)**({Y3}-1) * ( (1-TCR({Y3}+1))*((T-TY))
*( z**(1-1/sigma)*(1-alpha)*YQI**(1/sigma)-((SS_GGDP
/SS_POP)**({Y3}+(T-TY)))**(1-(T-TY))*WPRRA({Y3}+1)
**(T-TY)) )-((SS_GGDP/SS_POP)**({Y3}+(T-TY)))*(XF({Y3})
**(1-(T-TY))*XF({Y3}+1)**((T-TY))) )*(product(i=1 to {Y3}:
INFLE(i-1)/(1+(IS(i-1)+PREMK(i-1))/100)))) nu*(SS_GGDP
/SS_POP)*(INFLE-RES_INFLE)*XF(1)-(1-CR(1))*(SS_GGDP/SS_POP)
*WPRRA(+1)/(1+(IS+PREMK)/100)*RES_INFLE,
```

<u>(57) Age of the oldest production units in working order</u> (in years)

```
0=gsum{Y4}(((1-sign(AY-{Y4}-0.5)*sign(AY-{Y4}+0.5))/2)
```

```
*(log(INFL*(1-delta)/(1+(IS(-1)+PREMK(-1))/100))*(((A-AY)-
RES_A)*(1-TCR)*((SS_GGDP/SS_POP)**(-AY-(A-AY)+RES_A+1)
*YQI(-{Y4})-(WPRRA(-1)**(1-(A-AY)+RES_A)*WPRRA**((A-AY)-
RES_A)))-(XF(-1)**(1-(A-AY)+RES_A)*XF**((A-AY)-RES_A)) )
+(1-TCR)*((SS_GGDP/SS_POP)**(-AY-(A-AY)+RES_A+1)*YQI(-{Y4})-
(WPRRA(-1)**(1-(A-AY)+RES_A)*WPRRA**((A-AY)-RES_A)))-((A-AY)-
RES_A)*(1-TCR)*log(WPRRA/WPRRA(-1))*(WPRRA(-1)**(1-(A-AY)
+RES_A)*WPRRA**((A-AY)-RES_A))-log(XF/XF(-1))*(XF(-1)**
(1-(A-AY)+RES_A)*XF**((A-AY)-RES_A))-log(SS_GGDP/SS_POP)
*(((A-AY)-RES_A)*(1-TCR)*(WPRRA(-1)**(1-(A-AY)+RES_A)
*WPRRA**((A-AY)-RES_A))+(XF(-1)**(1-(A-AY)+RES_A)*XF**
((A-AY)-RES_A)))))
```

<u>(58) Aggregate supply of goods and services</u> (in millions of domestic currency at constant prices)

```
YQ = PT*(gsum{Y5}( ((1+sign(AY-{Y5}-0.5))/2) * ( (SS_GGDP
/SS_POP)**(-{Y5})*(1-delta)**({Y5})*YQI(-{Y5})*EPI(-{Y5}))
)+gsum{Y5}( ((1-sign(AY-{Y5}-0.5)*sign(AY-{Y5}+0.5))/2)
*(A-AY)*((SS_GGDP/SS_POP)**(-AY)*(1-delta)**( AY)*YQI(-{Y5})
*EPI(-{Y5})))) + RES_YQ
```

<u>(59) Aggregate employment</u> (in millions)

```
EP = gsum{Y5}( ((1+sign(AY-{Y5}-0.5))/2) * ( (1-delta)**
({Y5})*EPI(-{Y5}) ) )+gsum{Y5}( ((1-sign(AY-{Y5}-0.5)*sign
(AY-{Y5}+0.5))/2)*(A-AY)*( (1-delta)**(AY)*EPI(-{Y5})))
```

```
+RES_EP
```

III EQUATIONS OF THE STEADY STATE MODEL

III.1 National accounting identities (13 equations)

Goods and services market equilibrium, value

```
PMP= (PC*C+PA*(G+J+JG)+PC*WGRR*EG+PXM*EXMA-PMMA*IMM+
PCO_WO*(E/E90)*NXCA+PMP*SDVR)/GDP+PMP*RES_PMP
```

Value Added Tax

```
VAT= C*PC*VATR/(VATR+1)+PC*RES_VAT
```

GDP at factor costs

```
0 = log(((GDP-RES_GDP)*PMP-EG*PC*WGRR-VAT-PMP*VADR)/(YQ*P))
```

Goods and services market equilibrium, volume

```
GDP = C+G+(J-RES_J)+JG+PC*WGRR*EG/PMP + EXMA- IMM+NXCA+SDQ
```

Consumer price

```
LOG(PC)=pc1*LOG(P)+(1-pc1)*pc2*LOG(PMMA)+(1-pc1)*(1-pc2)*
LOG(E/E90*PCO_WO)+LOG((1+VATR)/(1+VATR90))+LOG(RES_PC)
```

Absorption price

```
PA=PC*(1+RPC)/(1+VATR)+PA*RES_PA
```

Receipts of general government

```
TT=VAT+(SCCR)*(P*WPRR*EP+PC*WGRR*EG)+TLR*(1-SCCR)*(P*WPRR
*EP+PC*WGRR*EG)+TCR*(P*YQ-(P*WPRR*EP))+OTR*PMP*GDP+PMP*RES_TT
```

Other taxes

```
OTR=otr0*OTR_EXO+(1-otr0)*(otr1*B1/(PMP*GDP)+otr2)+RES_OTR
```

Public balance (interest rates included)

```
PBAL = TT-(G+JG)*PA-EG*PC*WGRR-(PMP*TRR-BEN*P*(LF-ET))-
BEN*P*(LF-ET)-((1-BILG)*IS/100+BILG*IL/100)*B1+PMP*
```

```
ONDGR+PMP*RES_PBAL
```

<u>Net liabilities of general government</u>

```
B1 = (B1-PBAL)/(INFL*SS_GGDP)+(PMP/INFL)*RES_B1
```

<u>Trade balance, in current domestic currency</u>

```
TB = PXM*EXMA - PMMA*IMM + (E/E90)*PCO_WO*NXCA+PMP*RES_TB
```

<u>Net foreign assets, USD</u>

```
NFA1 = (((1-BILE)*(1+IS_US/100)+BILE*(IL_US/100+1))*NFA1A
+(TB/E+PMP_US*CTBR/E90))/(SS_INFL_US*SS_GGDP)+(PMP_US/SS_INFL
_US)*RES_NFA1
```

<u>Total employment</u>

```
ET = EP + EG + RES_ET
```

III.2 Households' consumption

<u>Unconstrained households' consumption</u>

```
0 = ((CU*(1-RES_CU))/L-cu0*CU/L*SS_POP/SS_GGDP)**(-cu1)-
cu2*(cu0+(1+IS/100)/SS_INFL)*(CU/L*SS_GGDP/SS_POP-cu0*(CU*(1-
RES_CU))/L)**(-cu1)+cu0*(cu2**2)*(1+IS/100)/SS_INFL*(CU/L
*(SS_GGDP/SS_POP)**2-cu0*CU/L*SS_GGDP/SS_POP)**(-cu1)
```

<u>Constrained households' consumption</u>

```
CC=cc0*((1-TCR)*(P*YQ-P*WPRR*EP)+(1-SCCR)*(1-TLR)*(P*WPRR*EP+
PC*WGRR*EG)+PMP*TRR)/PC+RES_CC
```

<u>Unconstrained households' consumption</u>

```
C=CU+CC+RES_C
```

III.3 Monetary policy

<u>Inflation rate of the GDP deflator</u>

```
INFLE = INFL+RES_INFL
```

<u>Interest rate term structure</u>

```
IL=IS+(1+100/IL)*RES_IL
```

Nominal short term interest rate US

```
1+IS_US/100 = SS_INFL_US*(SS_GGDP_US)**cu1_US/cu2_US +
is0_US*(INFL_US-SS_INFL_US) - premf0_US *SS_GGDP_US*SS_
INFL_US*(NFA1_US/(PMP_US*GDP_US))/100 + RES_IS_US/100
```

Nominal short term interest rate

```
1+IS/100 = SS_INFL*(SS_GGDP)**cu1/cu2 + is0*(INFL-SS_INFL)
+RES_IS/100
```

Uncovered interest rate parity

```
1+IS/100 - PREMF/100 = (SS_INFL / SS_INFL_US)*(1+IS_US/100) +
RES_E
```

Risk premiun on net foreign assets

```
PREMF = -premf0 *SS_GGDP*SS_INFL *(NFA1*E/(PMP*GDP))+SS_
INFL*RES_PREMF
```

Inflation of Consumer Prices

```
INFLC = SS_INFL +RES_INFLC
```

Nominal exchange rate ratio

```
DEP = 1 + RES_DEP
```

Net Foreign assets as % of GDP

```
NFA11 = SS_INFL*(NFA1*E/(PMP)) + RES_NFA11
```

Net Foreign assets as % of GDP, US

```
NFA11_US = SS_GGDP_US*SS_INFL_US*(NFA1_US/(PMP_US*GDP_US)) +
RES_NFA11_US
```

EURO ZONE

Risk premiun on net foreign assets

```
PREMF = -
premf0*SS_GGDP*SS_INFL*(NFA1*E/(PMP*GDP))+SS_INFL*RES_PREMF
```

Euro zone exchange rate parity

```
E = E_GE * E99/E99_GE + RES_E
```

Euro zone GDP prices

```
INFL = SS_INFL + RES_P
```

Uncovered interest rate parity

```
1+IS/100 - PREMF/100 = (SS_INFL / SS_INFL_US )* (1+IS_US/100)
+RES_IS
```

Uncovered interest rate parity EURO versus USD

```
1+IS_GE/100-PREMF_GE/100 =(SS_INFL_GE / SS_INFL_US )*
(1+IS_US/100) +RES_E_GE
```

BCE monetary policy

```
1+IS_GE/100 = SS_INFL_GE*(SS_GGDP_GE)**cu1_GE/cu2_GE +
is0_GE*(gprod{jj}(INFL{jj}**W_{jj})-SS_INFL_GE)+RES_IS_GE/100
```

where *jj* includes the euro area countries.

III.4 Foreign trade

Weighted average of foreign activity

```
FACTM = gsum{i}(SM_{i}*IMM_{i}/E90_{i}*E90)+RES_FACTM
```

where *i* includes the 17 modelled countries and the rest of the world.

Prices of competitors on export markets in domestic currency

```
LOG(PFM) = gsum{i}((WM_{i}/(1-SM_{i}))*gsum{k}(SM{k}_{i}*
LOG((PXM_{k})*(E/E_{k})/(E90/E90_{k}))))-LOG((PXM))*Gsum
{i}(WM_{i}*SM_{i}/(1-SM_{i}))+LOG(RES_PFM)
```

where *i* and *k* include the 17 modelled countries and the rest of the world.

Exports of manufactured goods and services (volume)

```
LOG(EXM/SS_GGDP)=LOG(FACTM/SS_GGDP)+exm3*Log((PFM)/(PXM))+((1
-exm1)*LOG(SS_GGDP)-exm0)/exms-(1/exms)*LOG(RES_EXM)
```

Export prices for manufactured goods in domestic currency

```
LOG(PXM/INFL)=((1-pxm3)*LOG((P)/INFL)+pxm3*LOG((PFM/INFL))
+((1-pxm1-pxm2)*LOG(INFL)-pxm0)/pxms)+LOG(RES_PXM**
(-1/pxms))
```

Real domestic economic activity

```
ACT= act1*C +act2*(J+JG) +act3*G +act4*EXM+ RES_ACT
```

Imports of manufactured goods (volume)

```
LOG(IMM)=(LOG(ACT)+imm4*LOG((PA)/(PMM))+((1-imm1)*LOG
(SS_GGDP)-imm0)/imms)+LOG(RES_IMM**(-1/imms))
```

Manufactured goods import price

```
LOG(PMM) = gsum{i}(SM{i}*LOG((PXM_{i})/(E_{i}/E90_{i})
*(E/E90)))+LOG(RES_PMM)
```

where *i* includes the 17 modelled countries and the rest of the world.

Net exports of primary commodities (volume)

```
NXC=nxc1*ACT+RES_NXC
```

World price of primary commodities, in current dollar, indexed to the US GDP-deflator

```
LOG(PCO_WO) = LOG(P_US)+LOG(RES_PCO_WO)
```

World trade discrepancy in manufactured goods and services, in constant dollars

```
WTMR_WO = (gsum{j}((IMM-EXM)/E90)+(IMM_RW-XM_RW*P_US/SS_P_US)
/E90_RW)+RES_WTMR_WO
```

where *j* includes the 17 modelled countries.

Export volume - manufactured goods, adjusted for real world trade discrepancy

```
EXMA = EXM+(WTSM/(1-WTSM_RW))*WTMR_WO*E90+RES_EXMA
```

Import price for manufactured goods, adjusted for nominal world trade discrepancy

```
PMMA = PMM-(WTSM/(1-WTSM_RW))*(gsum{j}(IMM*PMM/E-EXMA*PXM/E)
+IMM_RW*PMM_RW/E_RW-PXM_RW*EXM_RW*P_US/SS_P_US/E_RW)*E/
IMM+PXM/IMM*RES_PMMA
```

where *j* includes the 17 modelled countries.

World trade discrepancy for commodities, in constant dollars

```
WTCR_WO = gsum{i}((NXC_{i})/E90_{i})+ RES_WTCR_WO,
```

where *i* includes the 17 modelled countries and the rest of the world.

Export volume - primary commodities, adjusted for real world trade discrepancy

```
NXCA : NXCA = NXC-(WTSC/(1-WTSC_RW))*WTCR_WO*E90+ RES_NXCA,
```

Net foreign assets adjusted for world discrepancy, USD

```
NFA1A = NFA1 -(WTSM/(1-WTSM_RW))*(gsum{i}(NFA1_{i}))
+P/SS_INFL*RES_NFA1A,
```

where *i* includes the 17 modelled countries and the rest of the world.

III.5 Supply side blocks

Production function

```
YQI=z*RES_YQI*(alpha*KAPA**(1-1/sigma)+(1-alpha))**(sigma/
(sigma-1))
```

Investment excluding installation costs

```
JJ=PT*KAPA*(EPI-RES_EPI)
```

Goods and services market equilibrium, volume

```
J = ci0*(JJ-RES_JJ)
```

Real wage rate (wage curve)

```
log(WPRR) = wprr0 +log(YQ/EP)-prrs*log(SS_GGDP/SS_POP)+wprr1*
log(WDG)+wprr2*log(ET/LF)+((1-wprrs)*log(INFLE/INFL)+wprrs
*log(INFLE))+log(RES_WPRR)
```

Real wage rate, in efficiency unit

```
WPRR = (WPRRA-RES_WPRRA)*PT
```

Wedge

```
WDG = PC/((1-SCCR)*(1-TLR)*P)+RES_WDG
```

Expected life time of the new production units

```
0=log(INFLE*(1-delta)/(1+(IS+PREMK)/100))*(((T-TY)-RES_T)*(1-
TCR)*((SS_GGDP/SS_POP)**(-TY-(T-TY)+RES_T)*YQI-WPRRA)-XF)
+(1-TCR)*((SS_GGDP/SS_POP)**(-TY-(T-TY)+RES_T)*YQI-WPRRA)
-log(SS_GGDP/SS_POP)*(((T-TY)-RES_T)*(1-TCR)*WPRRA+XF),
```

Capital intensity of the new production units

```
z**(1-1/sigma)*alpha*((YQI/(KAPA-RES_KAPA))**(1/sigma))
*(gsum{Y3}( ((1+sign(TY-{Y3}+0.5))/2) * (INFLE*(1-delta)/
(1+(IS+PREMK)/100))**({Y3}) )+gsum{Y3}( ((1-sign(TY-{Y3}-
0.5)*sign(TY-{Y3}+0.5))/2) * (T-TY) * ((INFLE*(1-delta)/
(1+(IS+PREMK)/100))**({Y3}+(T-TY))))) = ci0/(1-TCR)
```

Factor cost frontier

```
gsum{Y3}( ((1+sign(TY-{Y3}+0.5))/2) * (INFLE*(1-delta)/
(1+(IS+PREMK)/100))**{Y3} * ( z**(1-1/sigma)*(1-alpha)
*YQI**(1/sigma)-((SS_GGDP/SS_POP)**{Y3})*WPRRA ) )
+ gsum{Y3}( ((1-sign(TY-{Y3}-0.5)*sign(TY-{Y3}+0.5))/2)*
(INFLE*(1-delta)/(1+(IS+PREMK)/100))**({Y3}+(T-TY)) * ((T-
TY)* ( z**(1-1/sigma)*(1-alpha)*YQI**(1/sigma)-((SS_GGDP
/SS_POP)**({Y3}+(T-TY)))*WPRRA)-((SS_GGDP/SS_POP)**({Y3}
+(T-TY)) )*XF/(1-TCR)) ) = -nu*(SS_GGDP/SS_POP)*(1-delta)*
(INFLE-RES_INFLE)*XF/(1-TCR) -(SS_GGDP/SS_POP)*(1-delta)*
WPRRA/(1+(IS+PREMK)/100)*RES_INFLE
```

Age of the oldest production units in woking order

```
0=log(INFL*(1-delta)/(1+(IS+PREMK)/100))*(((A-AY)-RES_A)*(1-
TCR)*((SS_GGDP/SS_POP)**(-AY-(A-AY)+RES_A+1)*YQI-WPRRA)
-XF)+(1-TCR)*((SS_GGDP/SS_POP)**(-AY-(A-AY)+RES_A+1)*YQI-
WPRRA)-log(SS_GGDP/SS_POP)*(((A-AY)-RES_A)*(1-TCR)*WPRRA+XF)
```

Aggregate supply of goods and services

```
YQ = PT*YQI*EPI*( gsum{Y5}( ((1+sign(AY-{Y5}-0.5))/2)
*( (SS_GGDP/(1-delta))**(-{Y5})) ) +(A-AY)*((SS_GGDP/(1-
delta))**(-AY)) )+RES_YQ
```

Aggregate employment

```
EP = EPI*( gsum{Y5}( ((1+sign(AY-{Y5}-0.5))/2)*((SS_POP/(1-
delta))**(-{Y5})))+(A-AY)*((SS_POP/(1-delta))**(-AY)))+RES_EP
```

IV PARAMETERS

SUPPLY SIDE PARAMETERS

Weighting capital parameter of the CES production function

alpha_AU	0.694014
alpha_BL	0.696708
alpha_CA	0.699504
alpha_DE	0.700599
alpha_FI	0.697199
alpha_FR	0.696681
alpha_GE	0.697527
alpha_GR	0.692533
alpha_IR	0.699561
alpha_IT	0.693267
alpha_JA	0.698085
alpha_NE	0.696791
alpha_PO	0.704405
alpha_SP	0.690464
alpha_SW	0.696521
alpha_UK	0.699653
alpha_US	0.697725

Unit cost of investment

ci0	1.25

Share of the labour on the new units of production coming from discarded units

nu	0
nxc1	-0.04

FAILURE RATES

delta	0.02

Capital-labour elasticity of substitution for the new production units

sigma	0.6

Consumption parameters

Consumer price parameters

pc1	0.8
pc2	0.8

cc0 – share of liquidity constrained households 0.13

cu0 – habit parameter

cu0_AU	0.74
cu0_BL	0.64
cu0_CA	0.20
cu0_DE	0.83
cu0_FI	0.63
cu0_FR	0.70
cu0_GE	0.63
cu0_GR	0.94
cu0_IR	0.64
cu0_IT	0.82
cu0_JA	0.97
cu0_NE	0.63
cu0_PO	0.69
cu0_SP	0.73
cu0_SW	0.39
cu0_UK	0.82

cu0_US	0.81
cu1 – time substitution parameter	0.84
cu2 – time preference parameter	0.96

Import propensity parameters for each demand component

act1	0.16
act2	0.33
act3	0.09
act4	0.15

Import equations parameters

imm0	0
imm1	0
imm2	0
imm3	1
imm4	0.5
imms	-0.3

Sensitivity of interest rate to net foreign assets

premf0	20

Estimated wage curve coefficients

wprr0	0
wprr1	0.15
wprr2	0.3
wprr2_AU	2.95
wprr2_FR	0.67
wprr2_GR	1.16
wprr2_NE	0.91
wprrs	0.74

Monetary rule parameters

is0	1.5

1990 GDP based weights in the monetary rule of ECB (for the EMU countries)

w_AU	0.031
w_BL	0.04
w_FI	0.026
w_FR	0.23
w_GE	0.289
w_GR	0.016
w_IR	0.01
w_IT	0.211
w_NE	0.055
w_SP	0.095

Fiscal rule parameters

otr0	0
otr1	0.15
otr2	0

Steady state growth rate of the GDP (plus 1)

SS_GGDP 1.015

Steady state growth rate of the population (plus 1)

SS_POP 1

Steady state growth rate of inflation (plus 1)

SS_INFL 1.015

1990 Nominal exchange rate vis-à-vis US$ (national currency/US$)

E99_AU	11.37
E99_BL	33.41
E99_FI	3.82
E99_FR	5.44
E99_GE	1.61
E99_GR	158.51
E99_IR	0.60

E99_IT	1198
E99_NE	1.82
E99_SP	101.9

FINAL APPENDIX 3

DATA AND BASELINE

This section describes the construction of the database needed to run the model Marmotte. It starts with a database filled with observed data as described in the note : DATA_NOTE.DOC (*f:\groupes\marmotte\donnees\data_note.doc*). This initial database is made of OECD data extracted with the AREMOS program: MARMOTTE.CMD (see note). The initial database spreads from 1970 to 1996 (it can be updated automatically by running MARMOTTE.CMD). Some data are non-available for this period. With this basic database, we first aim at extrapolating it with OECD medium-term forecasts (until 2005) and, second, to complete it in order to get a complete database useful for the model, i.e. spreading from 1910 to 2300. This second step gives the baseline of Marmotte. The last step before the simulation of the model concerns the calibration of the supply-side of the model and the computation of the residuals. This note presents these three steps and describes the programs needed to do so.

I COMPLETING THE INITIAL DATABASE WITH MEDIUM-TERM FORECASTS

This part presents the programs needed to complete the initial database with OECD medium-term forecasts. This is realised with two programs and executed with AREMOS. The inputs are the initial database (DATA.BNK) in AREMOS format, the OECD Economic Outlook database in AREMOS format (for short-term forecasts till 2001) and the OECD Medium-term forecasts in EXCEL format (for 2002-2005). The main program (EXTRAP.CMD) is made of two procedures (Extrap1 and Extrap2) and a main part piloting the procedures and an external program. This external program (TRADE.CMD) is devoted to the computation of trade variables. The output is a database (DATAMARMOTTE) in TSD format. It is used as the input of the second part.

I.1 First procedure: EXTRAP 1

The first procedure extrapolates the initial data over 1991-2001 using the OECD Economic Outlook. It first opens the specific country database

(f:\groupes\marmotte\donnees\wefa\baseau.bnk for Austria).

Each variable is extrapolated with an appropriate, available growth rate. Some variables are directly extrapolated from a similar series (e.g. The National Account GDP is extrapolated by using the OCDE Economic Outlook GDP growth rate). Some other variables are extrapolated by using their definition in Marmotte (the wedge is defined by the ratio between the consumption prices and the production prices corrected by tax rates available only in the initial database).

I.2 Second procedure: EXTRAP 2

The second procedure extrapolates the database obtained with the previous procedure over 2002-2005 by using the OECD Medium Term Scenario (EO67, June 2000). This database, initially in Excel format has been previously transformed in AREMOS databank by the program MT_OECD.CMD[79]. This database contains main growth rates (GDP, Exports, Imports, Prices…). After extrapolating some basic data, the procedure uses definition to calculate the data for all the variables.

I.3 Main program

The main program starts by opening the initial database (here DATA1.BNK; note that the name of this database depends on the options chosen in the program MARMOTTE.CMD see Note_Data.doc). It also opens the Economic Outlook database (PECO.BNK on the server I:\) and creates DATANEW.BNK.

The program copies the data from DATA1 into DATANEW and the trade data (EX, PX, IM and PM) from the specific country base (e.g. BASEAU).It opens the base TRADE.BNK to copy data f nominal exchange rates (E). This initialises the new database (message 'DATANEW initialised).

After this initialisation, the program launches EXTRAP 1 from 1991 to 2001 (Note that these two dates can be changed). Message 'EXTRAP1 done'.

Afterwards, it opens mt_oecd.bnk and launches EXTRAP 2 from 2000 to 2005 (Note that these two dates can be changed). Message 'EXTRAP2 done'.

Two lines are added to solve specific problems for Greece and Ireland (these could be discarded later after an updating).

I.4 Trade variables

A specific program (TRADE.CMD) realises the extrapolation of the trade variables. The main part of the program is similar to that devoted to trade data construction in the initial database. Some minor changes have been realised to construct this updated trade database. The output is a database named trade.bnk.

1.5 Exporting the new database

The end of the program creates the extrapolated database (Datamarmotte.bnk) from Datanew.bnk and trade.bnk. This new database is then exported in TSD format (readable by Troll). Message 'Datamarmotte data exported in TSD format'. It is stored in f:\groupes\marmotte\donnees\datamarmotte.tsd).

[79] The Medium-term scenario has been given by Pete Richardson (OECD).

II EXTENDING THE INITIAL DATABASE OVER THE DESIRED PERIOD (Datamarmotte.inp)

This second part presents the transformation of the previous updated database into the database of the model. The previous database (datamarmotte.tsd) covers the period 1970 to 2005 with some remaining non-available data for the 70s and the first half of the 80s. The complete database is obtained from a Troll program (datamarmotte.inp). This program includes two subprograms. The first one (mgparam.inp) creates the coefficient database. The second subprogram (extrap.inp), from the import of the initial TSD database, expands the TSD database until 2300 and retropolates it to the starting point (1910). After running these subprograms, datamarmotte.inp defines some variables, modifies some others and computes aggregated data for the EU and the Euro zone.

II.1 Creation of the coefficients database (mgparam.inp)

This program creates the coefficients database (named coefs.trl).

It defines some calibrated data (capital-labour elasticity of substitution for the new production units, failure rates, unit cost of investment, share of the labour on the new units of production coming from discarded units, firing cost relative to the annual wage rate).

It defines then coefficients derived from the estimations of the different relationships (wage curve coefficients, estimated wage curve coefficients, trade parameters, consumption parameters) and the calibrated values of the economic policy rules (fiscal rule parameters, sensitivity of interest rate to net foreign assets).

Finally, the program defines the steady-state growth rates (real growth, inflation, population).

II.2 Extrapolation and retropolation of the observed data (subprogram extrap.inp)

- Importing the data from the TSD database

The beginning of the program creates the new database in Troll format (datanew.trl). It imports the TSD database (datamarmotte.tsd) and copies it into the Troll database.

- Importing the growth rates (1962-2100) from a spreadsheet

There is a spreadsheet named retrop.wks which gives growth rates for real variables, prices and population. These growth rates are from the OECD economic outlook database and are extended until 2100 so as to reach at this date the steady-state growth rates.

The activity growth rates for the period 2005-2030 have been obtained from the work by Nina on the long-term scenario for growth (with Patrick Criqui – IEPE). These projections are based on a classical growth model. The long-term growth rate being around 1.5% in 2030, we have assumed that the steady-state growth rate is equal to 1.5%.

The population growth rates are from the UN population perspectives (2000 – 2050). After 2050, the population growth being negative for several countries, we have assumed that the populations stay constant (steady state growth rate equal to zero).

Inflation is assumed to be equal to 1.5% in the steady state. We have assumed a smooth adjustment between the last observed inflation rate (2005) to the steady-state growth rate (assumed to be in 2050).

After 2050 and till 2300, the growth rates are equal to their steady-state value.

- Extension of the database into the future

We class the variables into 4 categories (real variables, price variables, demographic variables and nominal variables). We extend each type of variables by the appropriate growth rate from 2005 to 2300.

- Retropolation 1910-last known date

We have identified for each variable of each country, the starting date of the series. According to the previous classification into 5 categories, we have defined different country-specific sub-categories relative to their starting date, in order to fill up the NA by values derived from the observed growth rates. The command also retropolates according to the assumed growth rates all the series so as to have data from 1910.

- The constant variables and wages

The constant variables are assumed to be set at their observed value in 1996 for the future and to their 1970 value for the past.

For wages, we assume that the wage of the private sector must represent a constant share in the value added (this share is assumed to be equal to 60%). We have rejected the idea to extend the wages with the last (or average) observed share in value added because of the 1990s seems to be a specific period for this share. This share determining the age of the oldest unit and the expected lifetime of the new units (see supply block), assuming a share equal across countries smoothes the differences for these values. The only difference across countries comes from the different corporate tax rates. This strong assumption enables us to have age and lifetime that are not diverging across countries in the long run and to have wage shares in value added that are not too inconsistent.

Finally, for the interest rates, they are assumed to be equal to their steady-state value (fixed according to the consumers' preference) from 2050 onwards. Between 1910 and 1970, interest rates are equal to the total period average (Troll function NAFILL). Between 2005 and 2050, interest rates are extrapolated so as the last observed value converged towards the steady-state according to a convergence speed given by the user (equal to 0.25).

II.3 Definition and aggregation (second part of Datamarmotte.inp)

The second part of Datamarmotte.inp starts by defining some variables :

Domestic activity, real wage rate in the public sector, different national account variables in real terms, inflation and expected inflation, investment after installation costs, world net foreign assets, adjusted net foreign assets and aggregated trade variables.

Then, it modifies some variables for Japan, Italy and Spain (million changed into billion)
Finally, it computes aggregated variables for the EU and the Euro zone.

III CALIBRATION OF THE SUPPLY-SIDE AND COMPUTATION OF THE RESIDUALS

This last part presents the calibration of the supply-side and the computation of the model's residuals.

III.1 Calibration of the supply-side

The unobserved supply-side variables are calibrated by a Troll program (Drivincal.inp). This calibration program enables us find values for the unobserved variables of the supply-side (YQI, TY, T, KAPA, PREMK, RES_WPRRA, RES_IS, EPI ,AY ,A ,alpha and z). The calibration is realised for the year 2010 (defined hare as a base year, any other year that does not exhibit unusual economic event – e.g. crisis, inflationary episode – could fit the calibration[80]). The calibrated model is solved for the unobserved variables for the base year and stored in a base named ccal.trl. The values for the base year are assumed to be invariant all over the period 1910-2300. The calibrated coefficients (alpha, z and tty) are stored in the coefficients database (coefs.trl). We also compute the coefficients for the aggregated zone (EU and EZ) as an average of the big countries of each zone.

III.2 Computation of the residuals

The computation of the residuals is realised by the program named resids.inp. The program called databases previously calculated (datanew.trl and ccal.trl). It computes some unobserved variables (constrained and unconstrained consumption), defines some others (technical progress, unemployment benefits) and initialises the residuals (1 for variables in log, calibrated values for WPRRA and IS, and 0 for the others). To calculate the residuals, we inverse the dynamic model (modnat.mod) such as the variables are all exogenous and the residuals are endogenous. The unconstrained consumption is only defined as an endogenous variable (because unobserved) and its residual is set to 0. Given the number of leads and lags, the dynamic model can be solved for the period 1995-2214. After the determination of the residuals, all the variables (residuals included) and parameters are stored in the baseline of Marmotte (ccss.trl).

[80] We could have been taken an average instead of a base year. Year 2010 seems to us both close to the last observations and not too far from the steady-state values.

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