Epidemics and fertility choices in a Malthusian economy

Abstract. This paper develops a framework to analyze the effects of an epidemic on fertility behavior. In a general equilibrium OLG model, adults and children consume each a specific good. The production of adult good requires land and labor as inputs. The children good is produced from labor only. Both goods enhance the expected lifetimes of those who consume them. Adults set their decisions to maximize the expected number of their children reaching the adult age.

An epidemic affects the numbers of workers and parents, their income, the intergenerational allocation of spending and the prices of goods. Consequently it changes the fertility rate of the population.

The model can reproduce the empirical result that the HIV/AIDS epidemic has led to a decrease in fertility in sub-Saharan Africa. However, it also suggests that this result is very sensitive to national conditions and the drop in fertility might be limited to one generation.

Keywords Fertility, Mortality, Epidemic, AIDS

JEL Classification J13, O41

Introduction

Young (2005, 2007) wrote two important empirical papers, which concluded that the AIDS epidemic in sub-Saharan Africa induced a decrease in the demand for children and in the fertility rate. In a Malthusian context this enhances per capita consumption. Young explains his result by a supply and cost mechanism. The care of children is more labor intensive than other economic activities. The epidemic has decreased the number of workers without affecting the quantities of the other factors (land, capital). Thus, it has led to higher wages of the living and consequently to a higher cost of children. Boucekkine and Debordes (2009) reach similar empirical results: a rise in adult mortality negatively influences fertility. On the other hand Juhn, Kalemli-Ozcan and Turan (2008) obtain less sanguine results. They find that local community HIV prevalence has no significant effect on non-infected women's fertility. Their paper also suggests that the effect of AIDS mortality on fertility could differ between countries.

The conclusions of Young are consistent with the demographic and economic history of medieval England. In 1316 the population of this country, at six million, was as great as in the early eighteenth century. But the arrival of the Black Death in 1348 is estimated to have reduced Britain's population to about half of its pre-plague level by 1377. It was followed by a long period of decline until 1450. By then England had barely two million people. Real wages increased threefold over this period (Clark, 2007). Young women could easily find a job and earn decent wages. Consequently, they became more independent of their family, could postpone the age of their marriage and have fewer children. On the other hand parents invested more in the human capital formation of their children. These features will last until the mid-16th century (de Moor and van Zanden, 2010). Young (2005) also believes that one of the consequences of the AIDS epidemic was to ease the access of women to the labor market, which strengthened their empowerment and decreased their fertility. These stylized facts are consistent with decreasing marginal return to labor and with a tradeoff between women's welfare and the fertility rate, induced by changes in the wage rate. They apply to the whole society and not only to its most affluent parts.

Nonetheless, Young's results are at odd with the history of the demographic transition (see for instance, Lorentzen, McMillan and Wacziark, 2009). In the demographic transition, mortality first falls and subsequently, fertility declines. This timing suggests that causality runs mostly from mortality to fertility rather than the reverse. Thus, the effects on fertility of a progressive and permanent change in mortality, seems very different from those of the sharp but transitory rise in deaths induced by an epidemic.

We develop in this paper a general equilibrium, overlapping generation model to clarify the transmission mechanisms of an epidemic to the fertility rate and the age structure of the population. Most of the theoretical literature on the effects of mortality on fertility behavior uses a Beckerian framework and assumes that parents have a tradeoff in their preferences between their own consumption, leisure, the number of their children and their quality (education, health, etc.). The strength of this approach is that it is very flexible and allows reproducing many situations of the reality. For instance, the explanation by Young (1995) of the negative effect of an epidemic on fertility is based on this approach. Becker and Barro (1988) use it to establish that a permanent decline in children's mortality should lead to a quantity – quality trade-off where parents have fewer children but invest more in each of them, which is what we see during the demographic transition. Boucekkine and Desbordes (2009) obtain the same result for a transitory decline in child mortality resulting from an epidemic.

This paper rests on the different assumption that, consistently with the traditional Darwinian theory, parents maximize the expected number of their children who will reach the adult age. If a parent consumes more, he improves his health and increases his expected lifetime and his capacity of having more children. Thus, when he sets his consumption and the number of his children if he remains alive, he just uses two different strategies targeted to the same end. This specification has the advantage of founding the behavior of parents on the developments of evolutionary biology (see Galor and Moav, 2002 and 2005, and Galor and Michalopoulos 2006, for earlier attempts to account for evolutionary biology ingredients in the theory of economic growth). One may find that the difference between the Beckerian and our objective function is overdone. The Beckerian framework is general and flexible enough to encompass our model, at a formal level at least. Nonetheless, it is precisely the specificity of our objective function and the fact that it has solid scientific foundations, which induced us to retain it. This paper tries to answer the question: how do epidemics change fertility when it is mainly determined by the desire of diffusing his genes?

Our paper also differs from most of the literature on death and fertility by assuming that children's consumption, which includes food, but also care, education, etc., enhances their probability of reaching the adult age. Thus, improving the quality of his children means rising their expected lifetime and not as in the Beckerian tradition, improving their education and their future income[1].

[1] De Moor and van Zanden (2010) and Young (2005) consider that men and women are imperfect substitute in productive activities and that the ratio of their respective wages changes with economic conditions. We have

Our analysis identifies the supply and cost effects introduced by Young. It also identifies income and demand mechanisms. Consider an epidemic like AIDS, hitting young adults. Those who survive will benefit from a higher quantity of factors, land and capital, and will be wealthier. This will induce them to having more children, and the fertility rate will increase. On the other hand, the death of young adults that is of people in the age to procreate will mechanically reduce the total number of children and so the demand for the labor intensive goods and services they consume. This will lower the wage rate and consequently the cost of children, which will enhance fertility still more.

Our model can reproduce the decrease in fertility, which Young observes in sub-Saharan Africa, for a specific kind of epidemic. It also shows that the effects of an epidemic on fertility are very sensitive to local condition: the different ages at which women bear children, men get married, people start working and people become infected. It also suggests that fertility could bounce back to its pre-epidemic value or even above in the time of one generation. This could explain a result by Juhn, Kalemli-Oczan and Turan (2008) that in high HIV countries such as Kenya and Lesotho, probably among the first to have been infected by the epidemic, the effect of community HIV prevalence on fertility of non-infected women is positive and significant. It is negative or mixed for the two lowest HIV countries in their sample, Niger and Senegal.

The model is presented in the first section. In the second section we successively compute its short run equilibrium, its steady state and its dynamics in the neighboring of the steady state.

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preferred to abstract from this important point and to remove all gender differences to make the analysis more tractable.

The last section is devoted to the analysis of two epidemics, one hitting children and the other hitting young adults.

1. The model

We consider a discrete time, perfect foresight dynamic model of a closed economy. People live for three periods, successively as children, young adults and senior adults.

1.1. The supply of goods

There are two goods in the economy. The production of the first good uses two factors, labor and land, with a Cobb-Douglas technology. The second good is produced with labor only and under constant returns. Young adults whose number in the current period, *t*, is N_t, are the only people who work. The total supply of land is normed to unity. Land is shared between the young adults in plots of equal size $1/N_t$. The tenants have no rent to pay, but they have to give their land back when they reach the age of senior adult so that it can be redistributed to the next generation of young adults[2]. Each young adult allocates his supply of labor, also normed to 1, between the production of the two goods, respectively l_t and $1-l_t$. His outputs of both goods are

(1) $x_t = Al_t^{\alpha}(1/N_t)^{1-\alpha}$, with A>0 and $0<\alpha<1$

(2) $y_t = 1-l_t$

The price of good y in terms of good *x*, which is the numeraire, is w_t. This price can also be interpreted as the wage rate. A young adult sets the allocation of his working time between the two productive activities to maximize his income $z_t = x_t + w_t y_t$. Let us define $\underline{w}_t = \alpha A / N_t^{1-\alpha}$. The outputs of the two goods and the income of the agent are respectively given by

(3) $x_t = A(\alpha A / w_t)^{\alpha/(1-\alpha)} / N_t$ if $w_t \geq \underline{w}_t$, $x = A / N^{1-\alpha}$ $x_t = \alpha A / N_t^{1-\alpha}$ if $w_t \leq \underline{w}_t$

(4) $y_t = 1 - \alpha x_t / w_t$ if $w_t \geq \underline{w}_t$, $y_t = 0_t$ if $w_t \leq \underline{w}_t$

(5) $z_t = w_t + (1-\alpha) x_t$ if $w_t \geq \underline{w}_t$, $z_t = x_t$ if $w_t \leq \underline{w}_t$

If the price of good *y* is not larger than $\underline{w}_t$, young adults specialize in the production of good *x*. Otherwise they produce both goods.

1.2. The demand of goods

The first good, *x*, is consumed by young adults and the second, *y*, is consumed by children. The consumption of these goods affects the lifetime of the agents, more precisely, it increases their probability of reaching their next stage of life. Good *x* aggregates food and all the other goods improving the quality of life and hygiene with of course healthcare goods. The second good also includes the time spent on the education and the care of children. Because of that it is more labor intensive than good *x*. We simplify the model and the presentation of our arguments by making the extreme assumption that good *y* is only produced with labor. We consider the choices of a young adult in the current period, *t*.

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[2] Galor and Moav (2002) and Galor and Michalopoulos (2006) instead assume that the rent on land is distributed to the workers in proportion to their labor, or what is equivalent, that workers are paid at their average and not at their marginal productivity.

This agent earns income z_t, consumes the quantity b_t of good *x* and stores the quantity s_t of this good (which can be stored without cost)

(6) $z_t = b_t + s_t$

His probability of being alive in the next period (*t*+1, as a senior adult) is an increasing function of his consumption of good *x*, $\pi(b_t)$. At the beginning of the next period the agent sets the number n_{t+1} of his children, if he is alive. Then, he sets the consumption c_{t+1} of good *y* by each of them. The probability for a child to be alive at the beginning of the following period, *t*+2, $\lambda(c_{t+1})$, depends on this consumption. As the price of good *y* is w_{t+1}, the spending of the agent is $n_{t+1}w_{t+1}c_{t+1}$. His saving is s_t. The bequest of young adults, who have died at the end of period *t*, is shared between all the senior adults living in period *t*+1 and spent on the consumption of their children. If we call r_t this supplementary income of a senior adult, the budget constraint of the agent in period $t+1$ is:

(7) $s_t + r_t = n_{t+1}w_{t+1}c_{t+1}$

Now, we shall give some justifications of the time structure of the life of an agent. We want to introduce in the model the feature that a person, by consuming more, can live longer and so have more children. In developing countries AIDS primarily affects young people in their child bearing age. When a person becomes HIV infected at the age of 25 he probably already has children, but he will have fewer children over his lifetime than a non infected person. We would have liked to assume that a senior adult works and gets the same wages as a young adult. However, this would have complicated the supply side of the model: the number of workers would have depended on the survival rate of young adults in the previous period. The simplest solution to take into accounts these objectives and constraints, is to split the lifetime of an adult into two periods, a first periods when he works and a second period when he has children.

The utility function of a young adult living in period *t* is

(8) $U_t \equiv \pi(b_t)\lambda(c_{t+1})n_{t+1}$

Our specification is in the spirit of evolutionary biology (Galor and Moav, 2002 and 2005, and Galor and Michalopoulos 2006). Consistently with the traditional Darwinian theory, a parent maximizes the expected number of his children reaching the age of young adult (he cannot influence the proportion of his children who will reach the age of senior adult, nor the number of their grandchildren). A parent can substitute a higher level of his own consumption to a higher fertility rate. However, he would do that only because by increasing his expected lifetime he improves his ability of having more children. Thus, there is no competition in our model between the selfish satisfaction for a parent of consuming more and his altruistic joy at having more children. Another specificity of our model is that the consumption of children directly affects their survival probability. In most theoretical models the investment in children's human capital that is in their quality has no effect on their probability of survival.

The survival functions of a child and of a young adult respectively are

(9) $\lambda(c_{t+1}) = \min\{C(c_{t+1} - C')^{1-\gamma}/(1-\gamma), 1\}$ if $c_{t+1} \geq C'$, $\lambda(c_{t+1}) = 0$ if $c_{t+1} \leq C'$

(10) $\pi(b_t) = \min\{B(b_t - B')^{1-\delta}/(1-\delta), 1\}$, if $b_t \geq B'$, $\pi(b_t) = 0$ if $b_t \leq B'$

with $0 < \gamma < 1$, $0 \leq \delta < 1$, $B, B', C, C' > 0$, $BB' < 1$, $(C'/\gamma)^{1-\gamma} C/(1-\gamma)^{\gamma} < 1$

We can see that a child will die with certainty if his consumption is not larger than *C'* and that a young adult will die with certainty if his consumption is not larger than *B'*. These two assumptions are similar to the fixed cost of raising a child, regardless of quality and to the subsistence consumption constraint, introduced by Galor and Moav (2002) in their model.

We deduce from equations (6) and (7) the expression of the number of children that the agent will have in period $t+1$, if he is still alive, in function of his consumption and the consumption of each of his children, b_t and c_{t+1}, and of his income in the two periods of his life, z_t and r_t

(11) $n_{t+1} = (z_t + r_t - b_t)/(w_{t+1}c_{t+1})$

The agent sets his consumption and the consumption of his children by maximizing the logarithm of his utility function

(12) $\log(U_t) \equiv \log[\pi(b_t)] + \log(z_t + r_t - b_t) + \log[\lambda(c_{t+1})] - \log(c_{t+1}) - \log(w_{t+1})$, with $b_t, c_{t+1} \geq 0$

We can see that the two choices of the agent are independent of the price of good *y*, w_{t+1}. We also see that the consumption of the agent maximizes the sum of the two first terms of the right-hand side of equation (12) and is independent of the survival function of the children. The consumption of each child maximizes the difference between the third and the fourth terms of the right-hand side and is independent of the survival function of the parent and of his income. Galor and Moav (2002) got a similar result. We have the two lemmas

Lemma 1. *The optimal consumption of good x and the probability of survival of a young adult are*

(13a) If $z_t \le B' - r_t$*, then* $0 \le b_t \le z_t + r_t$ *and* $\pi(b_t) = 0$

(13b) If $B' - r_t < z_t \le B' - r_t + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)}$,

then $b_t = [(1-\delta)(z_t + r_t) + B']/(2-\delta)$ *and* $\pi(b_t) = B(z_t + r_t - B')^{1-\delta} / [(2-\delta)^{1-\delta}(1-\delta^{\delta})]$

(13c) If $B' - r_t + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \le z_t$*, then* $b_t = B' + [(1-\delta)/B]^{1/(1-\delta)}$ *and* $\pi(b_t) = 1$

Proof. If $B' \le b_t \le B' + [(1-\delta)/B]^{1/(1-\delta)}$, we have

$$\log[\pi(b_t)] + \log(z_t + r_t - b_t) = (1-\delta)\log(b_t - B') + \log(z_t + r_t - b_t) + \log[B/(1-\delta)]$$

The first-order condition of the maximization of this expression relatively to b_t gives equation (13b).

If $[(1-\delta)(z_t + r_t) + B']/(2-\delta) \le B'$ that is if $z_t \le B' - r_t$, we have $0 \le b_t \le z_t + r_t$.

If $[(1-\delta)(z_t + r_t) + B']/(2-\delta) \ge B' + [(1-\delta)/B]^{1/(1-\delta)}$ that is if

$B' - r_t + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \le z_t$, we have $b_t = B' + [(1-\delta)/B]^{1/(1-\delta)}$.□

The consumption of a young adult is a non decreasing function of his income $z_t + r_t$. The scale parameters B has no effect on this consumption at the individual level that is when incomes, z_t and r_t, are given (except in the corner solution given by equation (13c)).

Lemma 2. *The optimal consumption of each child and his probability of survival are*

(14) $c_{t+1} = C'/\gamma$ *and* $\lambda(c_{t+1}) = (C'/\gamma)^{1-\gamma} C/(1-\gamma)^{\gamma}$

Proof. If $C' \le c_{t+1} \le C' + [(1-\gamma)/C]^{1/(1-\gamma)}$, we have

$$\log[\lambda(c_{t+1})] - \log(c_{t+1}) = (1-\gamma)\log(c_{t+1} - C') - \log(c_{t+1} - C') + \log[C/(1-\gamma)]$$

The first-order condition of the maximization of this expression relatively to c_{t+1} gives equation (14).

We can see that $c_{t+1} > C'$ and that $c_{t+1} \leq C' + [(1-\gamma)/C]^{1/(1-\gamma)}$ by using the conditions on the parameters of equation (9). □

The number of children, n_{t+1}, is given by equation (11). We deduce from lemmas 1 and 2 the lemma

Lemma 3. *The fertility rate is*

(15a) if $z_t \leq B' - r_t$, *there are no senior adults alive and so no children in period* $t+1$

(15b), if $B' - r_t < z_t \leq B' - r_t + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)}$, *then*

$$n_{t+1} = \gamma(z_t + r_t - B')/[(2-\delta)C' w_{t+1}]$$

(15c) if $B' - r_t + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \leq z_t$ *,then*

$$n_{t+1} = \gamma\{z_t + r_t - B' - [(1-\delta)/B]^{1/(1-\delta)}\}/(C' w_{t+1})$$

The fertility rate, n_{t+1}, is non decreasing in the income of the parent, $z_t + r_t$, and decreases with the price of good *y* that is the wage rate or the opportunity cost of labor in the period when children are born, w_{t+1}. We have made a strong distinction between the income of an agent, which increases the range of his choices, and the wage rate, which is also the price of the good consumed by children. This distinction is central in the paper by Galor and Michalopoulos (2006). It is especially clear in our model because these two variables intervene in different periods.

1.3. Markets equilibriums

In the current period *t*, the supply of good *x* is equal to its production, $N_t x_t$, plus the destoring by the senior adults and the bequests of the young adults who died at the end of the previous period, $N_{t-1}s_{t-1}$. The demand of this good is equal to the sum of its consumption and storing by the N_t young adults, $N_t(b_t + s_t)$. The equilibrium of the market of good *x* is

(16) $N_t x_t + N_{t-1}s_{t-1} = N_t(b_t + s_t)$

The supply of good *y* is equal to its production, $N_t y_t$. The demand of this good is equal to the consumption, c, of each of the $\pi(b_{t-1})N_{t-1}n_t$ children. The equilibrium of the market of good *y* is

(17) $N_t y_t = \pi(b_{t-1})N_{t-1}n_t c$

We remind that good *y*, includes the time spent on the care, education, etc., of children. We have assumed that there exists a market for this good. However, in our model, the parent is a senior adult who does not work. So, good *y* only includes the feeding, education and health services, produced by people other than the parents. This assumption may look artificial and we would have preferred to assume that senior adults work and get the same wage (which is the opportunity cost of raising children) as young adults. Nonetheless, as noted in paragraph 1.2 above, we have preferred not to make this last assumption because it would have complicated the supply side of the model and made the analysis less transparent.

Lemma 1 establishes that the consumption of each child, c, is positive. In the current period, *t*, variables b_{t-1}, N_{t-1} and n_t have predetermined values. If we assume that $b_{t-1} > B'$ and that the two other variables are positive, the demand for good *y* is positive. Thus, in equilibrium, the output of this good is also positive, which implies that its price satisfies the constraint $w_t > \underline{w}_t$[3]. We also notice that each senior adult in period $t+1$ gets a bequest from the young adults who died at the end of period t, equal to

(18) $r_t = s_t\left[1/\pi(b_t) - 1\right]$

If we add equation (16) to the product of equation (17) by the wage rate w_t, we get the relation

(19) $N_t(x_t + w_t y_t) + N_{t-1}s_{t-1} = N_t(b_t + s_t) + \pi(b_{t-1})N_{t-1}n_t w_t c$

We deduce from equations (4), (5) and (6) in period t that $N_t(x_t + w_t y_t) = N_t(b_t + s_t)$. Equations (7) and (18), written in period t-1, give $N_{t-1}s_{t-1} + N_{t-1}\left[1/\pi(b_{t-1}) - 1\right]s_{t-1} = N_{t-1}(s_{t-1} + r_{t-1}) = N_{t-1}n_t w_t c$. We can see that equation (17) is a linear combination of these two last equations. Thus, one of the two equilibrium equations (17) and (18) is redundant[4].

1.4 Demographic changes

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[3] If $b_{t-1} \le B'$ that is if $z_{t-1} + r_{t-1} \le B'$, then the demand for good y in period t is zero. Then, its production is also zero and his price satisfies the inequality $w_t \le \underline{w}_t$. However, in this case, all the young adults of period t-1 die at the end of the period and there are no senior adults living in period t. Consequently there are no births and no children living in this period. Finally, there are no more living people in period t+2. This scenario of doom could happen if an epidemic hit the young adults in period t-1 and rose the value of parameter *B'* above the maximum quantity of good x, which can be produced by a young adult $A(1/N_{t-1})^{1-\alpha}$.

[4] We assume that the saving of a young adult who dies before becoming a senior adult is redistributed between all the survivors. This assumption has some realism if the economy is composed of enlarged families with strong solidarity between its members. This social arrangement decreases the saving of each young adult: he knows that if he dies his saving will have been useless and that if he survives he will benefit of the saving of the dead, which is an incentive for him to save less. Consequently, this system of bequest increases the consumption of each young adult. It also implies that the market solution is suboptimal: if each young adult consumed less and saved more, he would have a higher expected number of children reaching the age of young adult.

The population in the current period, *t*, includes N_t young adults, $\pi(b_{t-1})N_{t-1}$ senior adults and $n_t\pi(b_{t-1})N_{t-1}$ children. In period *t*+1, it will include

(20) $N_{t+1} = n_t\lambda(c)\pi(b_{t-1})N_{t-1}$

young adults, $\pi(b_t)N_t$ senior adults, $n_{t+1}\pi(b_t)N_t$ children.

2. The solution of the model

We successively compute the short run equilibrium of the model, its steady state and finally its dynamics in the neighborhood of its steady state.

2.1 Computation of the short run equilibrium

The short run equilibrium in period *t* can be computed recursively. First, children's consumption and the probability of children's survival are the same in all periods and given by

(14) $c = C'/\gamma$ and $\lambda(c) = (C'/\gamma)^{1-\gamma}C/(1-\gamma)^{\gamma}$

The size of the population of young adults in the next period, N_{t+1}, is a function of variables, which are predetermined in the current period, that is the number of young adults and their probability of survival in period *t*-1, the fertility rate and the probability of survival of children in period *t*

(20) $N_{t+1} = n_t\lambda(C'/\gamma)\pi(b_{t-1})N_{t-1}$.

Lemma 4 establishes the expression of the current value of the price of good *y*, which is also the wage rate, w_t.

Lemma 4. *The current value of the price of good y,* w_t*, is a function of variables, which are predetermined in the current period,* $w_t = \varphi(s_{t-1}, N_{t-1}, N_t)$*. This function is implicitly defined by the equation*

(21) $w_t\left[1-(\alpha A/w_t)^{1/(1-\alpha)}/N_t\right]N_t = s_{t-1}N_{t-1}$

Proof. a) We substitute the value of *x* given by equation (5) in equations (3) and (16) and get

$$z_t = w_t + (1-\alpha)A(\alpha A/w_t)^{\alpha/(1-\alpha)}/N_t$$

$$z_t = w_t/\alpha - (1/\alpha - 1)s_{t-1}N_{t-1}/N_t$$

These two equations can be consistent only if w_t satisfies equation (21).

b) The left-hand side of equation (21) increases from zero to infinity when w_t increases from $\underline{w}_t$ to infinity. Thus, for any value of $s_{t-1}N_{t-1} \geq 0$ and $N_t > 0$, this equation determines a unique value of w_t.□

Equation (21) represents the equilibrium of the market of good *y*. The left-hand side is the value of the production of this good $w_t y_t N_t$. The right-hand side gives the demand for this good, which is equal to the saving of senior adults, $s_{t-1}N_{t-1}$. We can see that the current price of good *y*, which is also the wage rate, increases with the number of young adults in the previous period (they are the demanders for this good) and with their saving. It decreases with the number of young adults in the current period that is of the suppliers of this good. Then, we can use equations (5) and (16) to compute the value of the income of a young adult

(22) $z_t = \left[w_t - (1-\alpha)s_{t-1}N_{t-1}/N_t\right]/\alpha$

As we have just noticed, $s_{t-1}N_{t-1}/N_t$ is equal to the value of the production of good *y* by a young adult, $w_t y_t$. Then, if we use equation (4), we can see that equation (22) expresses that the current income of a young adult is equal to the value of his output $x_t + w_t y_t$.

Lemma 5 will give the expression of the consumption by a young adult, b_t, and of the number of his children, n_{t+1} in function of variables, which are predetermined in the current period or have been already computed

Lemma 5. *a) If* $B' < z_t \le B' + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)}$ *then* b_t *is the unique root of equation*

(23) $B(b_t - B')^{2-\delta}/(1-\delta)^2 + b_t - z_t = 0$.

The fertility rate is (24) $n_{t+1} = \gamma(b_t - B')/[(1-\delta)C'\varphi(s_t, N_t, N_{t+1})]$.

b) If $B' + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \le z_t$, *then (25)* $b_t = B' + [(1-\delta)/B]^{1/(1-\delta)}$ *and (26)* $n_{t+1} = \gamma\{z_t - B' - [(1-\delta)/B]^{1/(1-\delta)}\}/[C'\varphi(s_t, N_t, N_{t+1})]$.

c) If $z_t \le B'$, *then no young adult can survive and procreate in the next period. Thus, the population of young adults and the output of both goods will be zero in periods t+2 and the population will be zero in the following periods.*

Proof. a) If $B' - r_t < z_t \le B' - r_t + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)}$, equations (6) establishes that $s_t = z_t - b_t$. We deduce from equations (18) and (13b) that $r_t = (z_t - B')[1-\pi(b_t)]/[1-\delta+\pi(b_t)]$. We substitute this expression in equation (13b) and get equation (23). This equation has a unique root, if $z_t > B'$. Otherwise the equation has no root.

We deduce that $z_t + r_t - B' = [(2-\delta)/(1-\delta)](b_t - B')$. Thus, the constraints defining this case imply that $B' < b_t \leq B' + [(1-\delta)/B]^{1/(1-\delta)}$. We deduce from equation (15b) the expression of the fertility rate (24).

b) If $B' - r_t + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \leq z_t$, then $b_t = B' + [(1-\delta)/B]^{1/(1-\delta)}$ and $r_t = 0$. Thus, the constraint defining this case is $B' + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \leq z_t$. We deduce from equation (15b) the expression of the fertility rate (26).□

Finally, the expressions of the last variables, which determine the short-run equilibrium, are

(6) $s_t = z_t - b_t$,

(16) $x_t = z_t - s_{t-1} N_{t-1} / N_t$.

(27) $y_t = (s_{t-1} / w_t) N_{t-1} / N_t$

$\pi(b_t)$ is given by equation (10)

2.2. The steady state

Let us define *b'=b-B'*. The values of the variables in the steady state are given by the following proposition

Proposition 1. *a) If* $C'/(\gamma\lambda(C'/\gamma)) \leq 1/\{1 + \alpha(1-\delta) + \alpha(1-\delta)[B/(1-\delta)]^{1/(1-\delta)} B'\}$, *the model has a unique steady state such that b' is the positive root of equation*

$$(28)\ F(b') \equiv \left[1 - \frac{C'}{\gamma\lambda(C'/\gamma)}\right] B b'^{2-\delta} / (1-\delta)^2 - \frac{C'}{\gamma\lambda(C'/\gamma)} \alpha(b' + B') = 0$$

We also have $\pi(b')=Bb'^{1-\delta}/(1-\delta)$, $z=Bb'^{2-\delta}/(1-\delta)^2+b'+B'$, $s=Bb'^{2-\delta}/(1-\delta)^2$, $w=Bb'^{2-\delta}/(1-\delta)^2+\alpha(b'+B')$, $n=1/\left[\lambda(C'/\gamma)Bb'^{1-\delta}/(1-\delta)\right]$, *x=b'+B',* $y=s/w$ *and* $N=A(\alpha A/w)^{\alpha/(1-\alpha)}/x$.

b) If $1/\left\{1+\alpha(1-\delta)+\alpha(1-\delta)[B/(1-\delta)]^{1/(1-\delta)}B'\right\}\le C'/(\gamma\lambda(C'/\gamma))<1$, *the model has a unique steady state such that* $b'=[(1-\delta)/B]^{1/(1-\delta)}$, *x=b'+B',* $\pi(b')=1$, $z=\dfrac{1-(1-\alpha)C'/(\gamma\lambda(C'/\gamma))}{1-C'/(\gamma\lambda(C'/\gamma))}b$, $s=\dfrac{\alpha C'/(\gamma\lambda(C'/\gamma))}{1-C'/(\gamma\lambda(C'/\gamma))}b$, $w=\alpha b/[1-C'/(\gamma\lambda(C'/\gamma))]b$ $n=1/\lambda(C'/\gamma)$, $y=C'/(\gamma\lambda(C'/\gamma))$ *and* $N=A(\alpha A/w)^{\alpha/(1-\alpha)}/x$.

c) If $1\le C'/(\gamma\lambda(C'/\gamma))$, *the model has no steady state.*

Proof. See the appendix.

$C'/[\gamma\lambda(C'/\gamma)]=(C'/\gamma)^{\gamma}(1-\gamma)^{\gamma}/(\gamma C)$ is the average cost, measured in good *y*, of a child who reaches the age of young adult. Equation (28) expresses that this average cost, measured in good *x* (the *numeraire*), is equal to the saving of his parent. In the steady state the population must remain constant, so each young adult must produce, on average, one child who will reach the age of young adult. When $C'/[\gamma\lambda(C'/\gamma)]$ is higher than one there is no steady state because the output of good *y* cannot be larger than the maximum supply of labor per parent, which is one.

The fertility rate can be rewritten as $n=(1-\gamma)^{\gamma}(1-\delta)/\left[(C'/\gamma)^{1-\gamma}CBb'^{1-\delta}\right]$. In the next paragraph we will show that the steady state is stable for a wide range of values of the parameters. A permanent increase in child mortality can be formalized by an increase in the value of *C'* or a decrease in the value of *C*. We remind that on average each young adult must produce one child who will reach the age of young adult. This target does not change if child mortality increases, as in Becker and Barro (1988). For these authors this implies that the fertility rate has to increase to compensate for the higher child mortality. However, in our approach, a young adult can use the alternative strategy of increasing his consumption and so his expected lifetime and his opportunity of having children. We can easily establish that he will do that, if the value of *C’* increases or the value of *C* decreases. In the former case the cost C'/γ of bringing a child up increases too and each young adult will decrease his fertility rate (if he reaches the age of senior adult). In the latter case, the cost of bringing a child up remains the same and the fertility rate will decrease or increase according to the value of the parameters. Thus, it is only in this last situation that we can get the same result as Becker and Barro, which is consistent with what we observe during the demographic transition.

We can also easily show that a permanent increase in adult mortality that is a rise of *B’* or a decrease of *B*, still leads to an increase in the consumption of young adults. In the former case the fertility rate decreases, and in the latter case it will increase or decrease according to the values of the parameters.

2.3. The dynamics of the model in the neighborhood of the steady state

We limit our investigation of the dynamics of the model to case a of Proposition 1. Then, the probability for a young adult of reaching the age of senior adult is strictly included between zero and one. We deduce from equations (6), (20), (21), (22), (23) and (24) that the model can be reduced to a system of four equations, which determine the values of the variables w_t, $b_t^{'} = b_t - B'$, N_{t+1} and n_{t+1} in function of their lagged values

(29) $w_t\left[N_t-(\alpha A/w_t)^{1/(1-\alpha)}\right]=N_{t-1}Bb_{t-1}^{'2-\delta}/(1-\delta)^2$

(30) $N_{t+1}=n_t\lambda(C'/\gamma)N_{t-1}Bb_{t-1}^{'1-\delta}/(1-\delta)$

(31) $Bb_t^{'2-\delta}/(1-\delta)+b_t^{'}+B'=\left[w_t-(1-\alpha)(N_{t-1}/N_t)Bb_{t-1}^{'2-\delta}/(1-\delta)^2\right]/\alpha$

(32) $C'n_{t+1}w_{t+1}=\gamma b_t^{'}/(1-\delta)$

The linear approximation of equation (29) and (30) are

(33) $(1+\eta)\bar{y}\Delta w_t=(2-\delta)\bar{s}\Delta b_{t-1}/\bar{b}'-\bar{w}\Delta N_t/\bar{N}+\bar{s}\Delta N_{t-1}/\bar{N}$

(34) $\Delta N_{t+1}/\bar{N}=\Delta N_{t-1}/\bar{N}+\Delta n_t/\bar{n}+(1-\delta)\Delta b_{t-1}/\bar{b}'$

$\bar{w}$ is the steady state value of w_t and Δw_t is the differences between the current and the steady state values of this variable. We use the same notation for the other variables. $\eta=(\bar{w}/\bar{y})\partial\bar{y}/\partial\bar{w}=(\alpha A/\bar{w})^{1/(1-\alpha)}/\left[(1-\alpha)\bar{N}\bar{y}\right]=(1-\bar{y})/\left[(1-\alpha)\bar{y}\right]$ is the price elasticity of the supply of good *y*, computed at the steady state.

The linear approximation of equation (31) is $\alpha\left[(2-\delta)\bar{s}/\bar{b}'+1\right]\Delta b_t=\Delta w_t-(1-\alpha)\left[(2-\delta)(\bar{s}/\bar{b}')\Delta b_{t-1}^{'}+\bar{s}(\Delta N_{t-1}-\Delta N_t)/\bar{N}\right]$. We substitute the expression of Δw_t given by equation (33) and get

(35) $\Delta b_t/\bar{b}'=E_1\left[(2-\delta)\Delta b_{t-1}/\bar{b}'+\Delta N_{t-1}/\bar{N}\right]-E_2\Delta N_t/\bar{N}$, with

$E_1=\left\{1/\left[(1+\eta)\bar{y}\right]-(1-\alpha)\right\}(\bar{s}/\bar{b}')/\left\{\alpha\left[1+(2-\delta)\bar{s}/\bar{b}'\right]\right\}$ and

$E_2=\left\{1/\left[(1+\eta)\bar{y}^2\right]-(1-\alpha)\right\}(\bar{s}/\bar{b}')/\left\{\alpha\left[1+(2-\delta)\bar{s}/\bar{b}'\right]\right\}$

We have $1/\bar{y}^2>1/\bar{y}$, and $1/\left[(1+\eta)\bar{y}\right]=(1-\alpha)/(1-\alpha\bar{y})>1-\alpha$. This implies that $E_2>E_1>0$.

The right-hand side of equation (35) is proportional to the increase in the income of a young adult, Δz_t. If the number of young adults that is of *producers* in the current period, N_t, decreases, each of them can use a larger plot of land. Thus, his wage increases, but the rent rate on land decreases. The difference between these two changes in income, which is positive, is proportional to E_2. Thus, E_2 captures the supply effect, which is central to the analysis of Young (2005). If the number of young adults in the previous period, N_{t-1}, increases, the demand and the price of good *y* that is the wage rate in the current period increase too. However, because this change in demand also leads to a displacement of labor from the production of the land intensive good *x* to the production of good *y*, the rent rate on land decreases. The difference between these two changes in the income of a young adult in the current period, which is also positive, is proportional to E_1. Thus, E_1 catches the demand effect which was presented in the introduction. An increase in the saving of each young adult in the previous period, $s_{t-1} = Bb_{t-1}^{2-\delta} / (1-\delta)^2$, has the same effect as an increase in their number.

The linear approximation of equation (32) is $\Delta w_{t+1} / \overline{w} + \Delta n_{t+1} / \overline{n} - \Delta b_t / \overline{b}' = 0$. We substitute the value of Δw_{t+1} given by equation (33) and the values of $\Delta N_{t+1} / \overline{N}$ and Δb_t given by equations (34) and (35), and get

$$
\begin{aligned}
\frac{\Delta n_{t+1}}{\overline{n}} &= -\frac{1-(1-\delta-\eta)E_2}{1+\eta}\frac{\Delta N_t}{\overline{N}} + \frac{1/\overline{y}-(1-\delta-\eta)E_1}{1+\eta}\frac{\Delta N_{t-1}}{\overline{N}} \\
&+ \frac{(1-\delta)/\overline{y}-(2-\delta)(1-\delta-\eta)E_1}{1+\eta}\frac{\Delta b_{t-1}}{\overline{b}'} + \frac{1}{(1+\eta)\overline{y}}\frac{\Delta n_t}{\overline{n}}
\end{aligned}
\tag{36}
$$

The economic interpretation of the right-hand side of this equation is complex. Moreover, we cannot determine the signs of the effects of changes in the predetermined variables of the current period, on the fertility rate of the next period. For example, an increase in the population of young adults in the current period by ΔN_t, has three effects. First, as we have just noticed, it lowers their income. Thus, each young adult will save less and will be able to bear the cost of fewer children in the following period. This leads to a decrease in fertility. Secondly, each young adult will consume less and so have a lower probability of being alive in the following period and of being able to procreate. This will lead to a lower demand for good *y* and a lower price of this good in the following period. The cost of children will decrease, which will enhance fertility. Finally, a higher number of young adults in the current period implies a higher number of people who can procreate in the following period, and so a higher number of children, a stronger demand for good *y* and a higher price of this good. Thus, the cost of children will increase in the following period, which will decrease fertility.

Luckily, the specific features of an epidemic will allow us to reach more precise conclusions and a better interpretation of its effects on the fertility rate in section 3.

Finally, we can investigate the stability of the steady state. We have the lemma

Lemma 6. *The nonzero eigenvalues of the linear approximation of the model in the neighboring of its steady state are the roots of the equation in* Λ

$$(37)\quad F(\Lambda) \equiv \Lambda^2 - \left[(2-\delta)E_1 + \frac{1}{(1+\eta)\bar{y}}\right]\Lambda + \left[\frac{(2-\delta)E_1}{(1+\eta)\bar{y}} - \frac{\eta[1-(2-\delta)E_2]}{1+\eta}\right] = 0$$

Proof. The linear approximation of the model can be written in matrix form as

$$\begin{bmatrix} \Delta N'_{t+1}/\overline{N} \\ \Delta N_{t+1}/\overline{N} \\ \Delta b_t/\overline{b}' \\ \Delta n_{t+1}/\overline{n} \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ E_1 & -E_2 & (2-\delta)E_1 & 0 \\ \dfrac{1/\overline{y}-(1-\delta-\eta)E_1}{1+\eta} & -\dfrac{1-(1-\delta-\eta)E_2}{1+\eta} & \dfrac{(1-\delta)/\overline{y}-(2-\delta)(1-\delta-\eta)E_1}{1+\eta} & \dfrac{1}{(1+\eta)\overline{y}} \end{bmatrix} \begin{bmatrix} \Delta N'_t/\overline{N} \\ \Delta N_t/\overline{N} \\ \Delta b_{t-1}/\overline{b}' \\ \Delta n_t/\overline{n} \end{bmatrix}$$

Two of the eigenvalues of the matrix are zero. The two others are the roots of equation (37). □

Unfortunately, we could not get analytical results on the values of the roots of this equation. Thus, we computed their numerical values and drew their graph in function of $C'/[\gamma\lambda(C'/\gamma)] \in \,]0, 1/\{1+\alpha(1-\delta)+\alpha(1-\delta)[B/(1-\delta)]^{1/(1-\delta)}B'\}[$, for a large number of values of α (going from 0.1 to 0.9), δ (going from 0 to 0.9), *B* (going from 0.5 to 4) and *B'* (going from 0 to 0.2). The main result is that when *B'*=0, the eigenvalues have absolute values smaller than one. When *B'* increases, we keep this stability result for $C'/[\gamma\lambda(C'/\gamma)]$ larger than a lower bound, which decreases with *B* and increases with δ.

3. The effects of an epidemic

3.1. The various kinds of epidemics

There are major differences between epidemics. The Black Death randomly affected people across ages, while AIDS primarily affects young people in their child bearing age. The Spanish flu was deadly, unforeseen but short-lived. The specific features of an epidemic can have important implications for the dynamics of population, in particular for fertility, and this is what this paper is set out to study.

We define an epidemic hitting children as a decrease in the value of parameter C or an increase in the value of parameter *C'* of the survival function of children, lasting for one period. An epidemic essentially affects inherent health; nothing can be done against the epidemic itself although an increase in consumption will reduce the number of death toll. This reduction may be direct by dampening the effectiveness of the epidemic or indirect by decreasing the other causes of death. Longer epidemiological shocks would complicate tremendously the analytical treatment. A shock lowering C in the current period *t* will be called an epidemic of the *first kind* and a shock increasing C' will be called an epidemic of the *second kind*, as in Boucekkine and Laffargue (2010). We define the efficiency of children' consumption as the derivative of their probability of survival (9) with respect to their consumption $\partial\lambda(c_{t+1})/\partial c_{t+1} = C(c_{t+1} - C')^{-\gamma}$. We have $\partial^2\lambda(c_{t+1})/(\partial c_{t+1}\partial C) = (c_{t+1} - C')^{-1-\alpha} > 0$, $\partial^2\lambda(c_{t+1})/(\partial c_{t+1}\partial C') = C\gamma(c_{t+1} - C')^{-1-\gamma} > 0$ and $\partial^2\lambda(c_{t+1})/\partial c_{t+1}^2 = -C\gamma(c_{t+1} - C')^{-1-\alpha} < 0$. The efficiency of consumption *decreases* with an epidemic of the first kind. It *increases* with an epidemic of the second kind. This efficiency decreases with the level of consumption, which is a reasonable result.

Lemmas 2 and 3 show that *if we keep everything else constant* (that is the income of each parent, $z_t + r_t$), an epidemic of the first kind has no effect on the children's consumption, decreases their survival probability and has no effect on the fertility rate of parents. An epidemic of the second kind (which rises the efficiency of consumption), leads to an increase in the consumption of each child that is to a higher cost of children, an *increase* in his survival rate and a decrease in the fertility rate of parents. This last result is consistent with a result by Galor and Moav (2002), which is that a rise in the return on human capital leads to invest more in each child and to have fewer children. These epidemics have no effect on the consumption of parents or on their survival probabilities.

The same considerations can be made on an epidemic affecting young adults that is lowering parameters B or increasing parameter B' in period *t*. The efficiency of young adults' consumption is $\partial\pi(b_t)/\partial b_t = B(b_{t-1} - B')^{-\delta}$. It decreases with an epidemic of the first kind and increases with an epidemic of the second kind. Lemmas 1 and 3 show that, if we keep everything else constant, an epidemic of the first kind has no effect on the consumption and the fertility rate of young adults. It decreases their survival probability. On the other hand, an epidemic of the second kind leads to an increase in the consumption of young adults and to a decrease in their fertility rate[5]. Their survival probability still decreases. These epidemics have no effect on the children's consumption and on their survival probabilities.

When we define an epidemic we have to be precise on the time when people understand its nature. In this section we assume that parents know the existence of an epidemic hitting children *before* the birth of their children. Thus, they can adjust their fertility rate and the consumption of each child. Young adults know that there is an epidemic hitting them at the

[5] We could have assumed instead that a parent maximizes the *expected value of a concave function of the number of his children* reaching the age of young adult. Then, we would introduce a precautionary demand for children. The usual result in the theoretical literature is that in the presence of this effect, an increase in mortality will lower the investment in each child and rise fertility (see, for instance Kalemli-Ozcan (2006), and Juhn, Kalemli-Ozcan and Turan (2008) in the context of the AIDS epidemic in sub-Saharan Africa). However, our model differs from this literature by assuming that the investment in a child increases his survival rate. Then, we can show that the precautionary motive leads to more investment in each child, less consumption by his parent and a lower fertility rate. The effect of an epidemic of the second kind, hitting children or young adults, on fertility, is still negative, but weaker than without the precautionary motivation.

beginning of this stage of life that is before making their consumption and labor allocation decisions.

We will assume that before the epidemiological shock, the economy was in a steady state equilibrium of the a type of Proposition 1[6]. We will first consider the effects of an epidemic of the first kind hitting children in period *t*, that is of a negative change in the value of parameter *C* by $\Delta C < 0$. Then, we will examine the effects of the same kind of epidemic hitting young adults.

3.2. An epidemic of type 1 hitting children

We have the proposition

Proposition 2. An epidemic of type 1 hitting children in period t has no effect on the fertility rate in the same period. It leads to a decrease in the fertility rate and in the number of young adults in period t+1. In period t+2, the fertility rate decreases (increases) if $1/[(1+\eta)\bar{y}]^2 + (1-\delta)E_2 > (<) 1$*, and the number of young adults decreases*

Proof. See the appendix.

We saw in the previous paragraph that this epidemic has no effect on the consumption of children or on the fertility rate of parents. However, the proportion of children who will be alive in the next period, *t*+1, that is the number of young adults and the supply of labor in this

[6] This assumption is probably unrealistic in the case of sub-Saharan Africa and the AIDS epidemic, although it was probably valid in the case of the Black Death. However, if the model were linear, the results of the two next paragraphs would still be valid as describing the difference between the paths followed by the demographic and

period, will be lower. As the saving of the young adults in the period of the epidemic, *t*, has not been affected, the demand of good *y* in period *t*+1 is unchanged. Consequently, the price of this good (which is also the wage rate) increases and the fertility rate decreases in this period.

As the income of the young adults of period *t*+1 will be higher, they will be able to bear the cost of more children in period *t*+2, which will enhance their fertility rate. As a higher proportion of them will be alive in period *t*+2 and as each of them will spend more on good *y*, the price of this good will increase, which will dampen the fertility rate. On the other hand, as the fertility rate has decreased in period *t*+1, there will be fewer young adults and workers alive in period *t*+2. This will lead to a lower supply of good *y* and to an increase in its price, which will dampen more the fertility rate. Thus, the sign of the change of the fertility rate in period *t+2* is undetermined. The proposition includes a condition, which determines this sign but which cannot be given a simple economic interpretation.

We must clarify a point of terminology. An epidemic hitting children in period *t* kills them when they reach the age of young adult, *after* that their parents have born the cost of their upbringing. The numbers of young adults and workers in period *t*+1, decrease. Thus, the supply and cost effect considered by Young (2005) prevails in this period and we are not surprised to obtain a drop in the fertility rate, as this author. In period *t*+2, we still have a supply effect, which tends to depress the fertility rate. However, we also have a series of income and cost demand mechanisms, which have uncertain effects on this rate. Boucekkine and Desbordes (2009), instead, assume that the epidemic kills children when they are very

economic variables with and without epidemic. This is still approximately the case with our nonlinear model in the neighborhood of the steady state.

young (before the age of five in the empirical part of the paper) that is before that the parents have spent much on them. Thus, parents react to an epidemic by keeping their net fertility rate unchanged and by increasing their total fertility rate by the amount necessary to compensate for the dead children. Our model reaches the same result as Boucekkine and Desbordes with their definition of an epidemic. These authors empirically confirm their theoretical result on data from sub-Saharan Africa.

To illustrate the theoretical results we ran a simulation of the model. We set the values of its parameters to $\alpha = 0.5$, $B = 2.5$, $B' = \delta = 0$ and $C'/(\gamma\lambda(C'/\gamma)) = 0.6$, without pretending that these values have empirical foundations. In the steady state we have $\bar{b} = \bar{x} = 0.3$, $\bar{\pi} = 0.75$, $\bar{y} = 0.6$, $\bar{w} = 0.375$. The two eigenvalues of the model are equal to 0.0982 and 0.8733. The epidemic is a decrease in the value of parameter C by 20 per cent in period 1. As, we have $1/[(1+\eta)\bar{y}]^2 + (1-\delta)E_2 = 0.9425 < 1$, the fertility rate increases in period 3. The results of the simulation are given in Figure 1. We can see the decrease, then the increase in the fertility rate. We can also see that the size of the population drops in the year of the epidemic, then slowly comes back to its initial level. In period 2 the number of young adults decreases by 20%, the number of children by 14.3% and the number of senior adults is unchanged. In period 3, the sizes of these three age-cohorts respectively are 14.3%, 10.2% and 11.71% under their initial values. This evolution of the age pyramid over time is a little more complex than a progressive translation of the trough in the population of young adult induced by the epidemic, to older generations (this move can be observed for instance in the projections of the 2004 United Nation report) because our model endogeneizes the expected lifetime end fertility.

HERE FIGURE 1

3.3. An epidemic of type 1 hitting young adults

We have the proposition

Proposition 3. An epidemic of type 1 hitting young adults in period t has no effect on the fertility rate of the period. It leads to an increase in the fertility rate and to a decrease in the number of children and senior adults in period t+1. In period t+2, the fertility rate and the sizes of the three age cohorts decrease if $\frac{\eta}{1-\eta-\delta}\frac{1}{\bar{y}} > \frac{\bar{\pi}}{(2-\delta)\bar{\pi}+1-\delta}$. *Otherwise, the fertility rate and the size of the child population increase; the sizes of the young and senior adult population decreases.*

Proof. See the appendix.

We saw in the paragraph 3.1 that this epidemic would have no effect on the consumption and saving of young adults if their income, including the bequests obtained by those who will reach the age of senior adult, were unchanged. However, young adults know that a lower proportion of them will be alive in the next period and that they will benefit from higher bequests. These expectations induce young adults to save less and consume more in the current period. This pushes down the demand of good *y* in the following period.

On the other hand the number of workers that is of young adults in period t+1 is unaffected by the epidemics (these people were children in period t). Thus, the price of good *y* and consequently the cost of a child in period t+1, decreases. Even if each young adult saves less in period t, those who survive in period t+1 are fewer, they receive more bequests and their income is higher. This rise of income and the reduction in the cost of children lead to an increase in the fertility rate in period t+1. However, this increase is insufficient to prevent a drop in the number of children in this period.

An epidemic hitting young adults in period t kills them when they reach the age of senior adult. Thus, the epidemic has no effect on their production in period t, but decreases the number of senior adults in period t+1 and consequently the number of births in this period. We have a series of income and cost demand effects in period t+1, without supply effect, which lead to a higher fertility rate.

The income of young adults in period t+1 decreases because of their lower wage rate. Thus, these agents consume and save less. Consequently they have a lower probability of being alive in period t+2. Their lower saving leads to a lower demand for good y in period t+2. On the other hand, we saw that the number of children has decreased in period t+1. Thus, the number of young adults that is of workers, alive in period t+2, is lower. This leads to a decrease in the supply of good y in this period. We establish in the proof that the supply effect is stronger than the demand effect and that the price of good y increases, if $\frac{\eta}{2-\delta}\frac{1}{\overline{y}} > \frac{\overline{\pi}}{(2-\delta)\overline{\pi}+1-\delta}$. This and the decrease in the saving of young adults in period t+1 lead to a lower fertility rate in period t+2.

We ran a simulation of the model for the same values of the parameters as in the previous paragraph. The epidemic is a decrease in the value of parameter B by 20 per cent in period 1. The relative changes in the fertility rate and in the sizes of the three age cohorts are given in Figure 2. We can see on the graphs the rise, then the decrease in the fertility rate, the strong fall of the number of senior adults and the smaller decrease in the number of children and young adults in the period, which follows the epidemic.

HERE FIGURE 2

We have assumed in this paragraph that the epidemic was fully expected by the young adults at the beginning of period *t*, before they make their consumption, saving and fertility decisions. If the epidemic takes place at the end of the period when the young adults have already made their decisions and cannot change them, the analysis becomes simpler. Then, the number of senior adults and children decreases in period *t*+1. So, the number of young adults decreases in period *t*+2. Consequently, the wage rate and price of good *y* increases. Finally, the fertility rate decreases in period *t*+2. Proposition 3 can be interpreted as applying to long lived epidemics, like AIDS, to which people have time to adjust their behavior. Our last considerations apply to short and unforeseen epidemics, like the Spanish flu.

Conclusion

We have built a theoretical general equilibrium model to investigate the effects of an epidemic on fertility. The model makes two distinctive assumptions. First, the agents make their decisions to maximize the expected number of their children who will reach the adult age. Second, parents allocate their income between their children's consumption, which improves their probability of reaching the adult age, and their own consumption which extends their expected lifetime and so their capacity of having more children.

The model can generate the dynamic paths of the fertility rate and of the size and age pyramid of the population after an epidemiological shock. If an epidemic lowers the proportion of children who reach the adult age that is the population of people who work, we obtain the same conclusion as Young in the short run: a rise in the wage rate and a decrease in fertility. However, as the number of people who can procreate has become smaller the total demand of children is lower, which will lower the wage rate and the cost of children. Moreover people who have survived the epidemic are wealthier. Thus, in the medium run the fertility rate may bounce back to its pre-epidemic level or even above. Then, the gift of the dying would be more short-lived in the sub-Saharan Africa of nowadays than in medieval England.

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APPENDIX

Proof of Proposition 1

The steady state of the model is defined by the system of equations

(20) $1 = n\lambda(C'/\gamma)Bb'^{1-\delta}/(1-\delta)$

(21) $w\left[1-(\alpha A/w)^{1/(1-\alpha)}/N\right] = s$

(22) $\alpha z = w-(1-\alpha)s$

(6) *s=z-b'-B'*

(16) $x = b'+B'$

(27) *y=s/w*

a) If $B' < z \le B'+[(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)}$,

(23) $Bb'^{2-\delta}/(1-\delta)^2 + b'+B'-z = 0$, with *b'>0*

(24) $n = \gamma b'/[(1-\delta)C'w]$

b) If $B'+[(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \le z$,

(25) $b' = [(1-\delta)/B]^{1/(1-\delta)}$

(26) $n = \gamma\left\{z-B'-[(1-\delta)/B]^{1/(1-\delta)}\right\}/(C'w)$

a) In the first regime we express all variables in function of *b'*. Then, we compute the value of *b'*.

We have $z = Bb'^{2-\delta}/(1-\delta)^2 + b'+B'$. Thus, the conditions of the regime,

$B' < z \le B'+[(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)}$, are equivalent to

$0 < b'\left[Bb'^{1-\delta}/(1-\delta)^2+1\right] \leq \left[(2-\delta)/(1-\delta)\right]\left[(1-\delta)/B\right]^{1/(1-\delta)}$. We also have $\pi(b') = Bb'^{1-\delta}/(1-\delta)$ and $s = z - b' - B' = Bb'^{2-\delta}/(1-\delta)^2$. Thus, $w = \alpha z + (1-\alpha)s = Bb'^{2-\delta}/(1-\delta)^2 + \alpha b' + \alpha B'$ and $n = \dfrac{1}{\lambda(C'/\gamma)b'^{1-\delta}/(1-\delta)} = \dfrac{\gamma}{(1-\delta)C'w}b' = \gamma\dfrac{b'}{(1-\delta)C'\left[Bb'^{2-\delta}/(1-\delta)^2+\alpha b'+\alpha B'\right]}$. Thus, $\dfrac{(1-\delta)C'}{\gamma\lambda(C'/\gamma)}\left[Bb'^{2-\delta}/(1-\delta)^2+\alpha b'+\alpha B'\right] = BBb'^{2-\delta}/(1-\delta)$. Finally, we get equation (28).

We have $F(0) = -C'/[\gamma\lambda(C'/\gamma)] < 0$. $F'(b') \equiv \left[1-\dfrac{C'}{\gamma\lambda(C'/\gamma)}\right]\dfrac{2-\delta}{(1-\delta)^2}Bb'^{1-\delta} - \dfrac{(1-\delta)C'}{\gamma\lambda(C'/\gamma)}\alpha$. If $C'/[\gamma\lambda(C'/\gamma)] < 1$, then $F'(b')$ has a unique positive root. $F(b')$ increases indefinitely with b'. So, $F(b')$ has a unique root, which is positive. Otherwise, it has no root.

The condition for the regime is $Bb'^{2-\delta}/(1-\delta)^2 + b' - [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)} \leq 0$. The left-hand side of this inequality increases indefinitely from a negative value when b' increases from zero to infinity. It has a unique root $[(1-\delta)/B]^{1/(1-\delta)}$. So, the condition is equivalent to $b' \leq [(1-\delta)/B]^{1/(1-\delta)}$. The root of $F'(b')$ satisfies this constraint if $F\left\{[(1-\delta)/B]^{1/(1-\delta)}\right\} \leq 0$.

We also have $x=b'+B'=b$ and $N = A(\alpha A/w)^{\alpha/(1-\alpha)}/x$. We deduce that $(\alpha A/w)^{1/(1-\alpha)}/N = 1-y = \alpha x/w = \alpha b/\left[B(b-B')^{2-\delta}/(1-\delta)+\alpha b\right] < 1$. Hence $w > \underline{w} = \alpha A/N^{1-\alpha}$.

b) In the second regime we have $n = 1/\lambda(C'/\gamma)$, $b = B' + [(1-\delta)/B]^{1/(1-\delta)}$ and $\pi(b) = 1$. We also have $n = \gamma\left\{z - B' - [(1-\delta)/B]^{1/(1-\delta)}\right\}/(C'w)$. Thus, $wC'/(\gamma\lambda(C'/\gamma)) = z - b$. Moreover $w = z - (1-\alpha)b$. Finally, $[z-(1-\alpha)b]C'/(\gamma\lambda(C'/\gamma)) = z - b$

that is $z = \dfrac{1-(1-\alpha)C'/(\gamma\lambda(C'/\gamma))}{1-C'/(\gamma\lambda(C'/\gamma))}\left\{B' + [(1-\delta)/B]^{1/(1-\delta)}\right\}$.

The condition $z \geq B' + [(2-\delta)/(1-\delta)][(1-\delta)/B]^{1/(1-\delta)}$ becomes

$C'/(\gamma\lambda(C'/\gamma)) \geq 1/\left\{1+\alpha(1-\delta)+\alpha(1-\delta)[B/(1-\delta)]^{1-\delta}B'\right\}$.

We also have $w = \frac{\alpha}{1 - C'/(\gamma\lambda(C'/\gamma))}\left\{B' + [(1-\delta)/B]^{1-\delta}\right\}$,

$s = \frac{\alpha C'/(\gamma\lambda(C'/\gamma))}{1 - C'/(\gamma\lambda(C'/\gamma))}\left\{B' + [(1-\delta)/B]^{1-\delta}\right\}$, $x = \left\{B' + [(1-\delta)/B]^{1-\delta}\right\}$ and $N = (\alpha A/w)^{1/(1-\alpha)}/x$. w, z and s will be positive if $C'/(\gamma\lambda(C'/\gamma)) < 1$. We easily check that $w > \underline{w} = A\alpha/N^{1-\alpha}$. □

Proof of Proposition 2

We compute the new short run equilibrium of the model successively in periods t and t+1, as explained in paragraph 2.1.

a) In period t, parameter C changes by $\Delta C < 0$. According to equation (14), the consumption of each child born in the period is unchanged, but his probability of being alive in period t+1 decreases by $\Delta\lambda_{t+1}/\bar{\lambda} = \Delta C/C < 0$. The saving of the parent of this child is unchanged, as the number of young adults, $\bar{N}$. Thus, according to equations (21) the price of the y good, $\bar{w}$, does not change. According to equation (7) the fertility rate of the senior adults in the period, $\bar{n}$, does not change. The size of the population of young adults in period t+1 decreases by $\Delta N_{t+1}/\bar{N} = \Delta C/C < 0$.

According to equations (22) and (23) the income, the consumption, $\bar{b}$, and the probability of being alive in the following period, $\pi(\bar{b})$, of the young adults in period t does not change. According to equation (6) their saving, $\bar{s}$, does not change either. Thus, we deduce from equation (21) that the price of good y is higher in period *t+1* by $\Delta w_{t+1}/\bar{w} = -[1/(1+\eta)\bar{y}]\Delta N_{t+1}/\bar{N} = -[1/(1+\eta)\bar{y}]\Delta C/C > 0$. Finally, equation (24) implies that the fertility rate in period t+1 decreases by $\Delta n_{t+1}/\bar{n} = [1/(1+\eta)\bar{y}]\Delta C/C < 0$.

b) According to equation (20) the number of young adults in period *t+2* decreases by $\Delta N_{t+2}/\bar{N} = \Delta n_{t+1}/\bar{n} = (\Delta C/C)/[(1+\eta)\bar{y}] < 0$. According to equation (22) the income of a young adult in period t+1 increases by $\Delta z_{t+1} = -\bar{b}'\left[1+(2-\delta)\bar{s}/\bar{b}'\right]E_2\Delta C/C > 0$. According to equation (23) the consumption of this young adult and his probability of reaching the age of senior adult increase by $\Delta b_{t+1}/\bar{b}' = -E_2\Delta C/C > 0$ and $\Delta\pi_{t+1}/\bar{\pi} = -(1-\delta)E_2\Delta C/C > 0$. According to equation (6) the saving of this young adult increases by $\Delta s_{t+1}/\bar{s} = -(2-\delta)E_2\Delta C/C > 0$. Then, according to equation (21) the price of good y in period t+2 changes by

$$\frac{\Delta w_{t+2}}{\bar{w}} = -\frac{1}{\bar{y}(1+\eta)}\frac{\Delta N_{t+2}}{\bar{N}} + \frac{\Delta s_{t+1}}{\bar{s}} + \frac{\Delta N_{t+1}}{\bar{N}} = \left[1 - \frac{1}{(1+\eta)^2\bar{y}^2} - (2-\delta)E_2\right]\frac{\Delta C}{C}.$$

Finally, we deduce from equation (24) that the fertility rate in period t+2 changes by

$$\frac{\Delta n_{t+2}}{\bar{n}} = -\left[1 - \frac{1}{(1+\eta)^2\bar{y}^2} - (1-\delta)E_2\right]\frac{\Delta C}{C}. \square$$

Proof of Proposition 3.

We compute the new short run equilibrium of the model successively in periods t and t+1, as explained in paragraph 2.1.

a) In period t, parameter B changes by $\Delta B < 0$. Equation (23) shows that the consumption of a young adult increases by $\Delta b_t/\bar{b} = -(\Delta B/B)\bar{\pi}/\left[(2-\delta)\bar{\pi}+1-\delta\right] > 0$. His survival probability decreases by $\Delta\pi_t/\bar{\pi} = (\Delta B/B)(\bar{\pi}+1-\delta)/\left[(2-\delta)\bar{\pi}+1-\delta\right] < 0$. Equation (6) shows that the saving of a young adult decreases by the same amount as his consumption increases. Thus, $\Delta s_t/\bar{s} = -(\bar{b}'/\bar{s})\Delta b_t/\bar{b}' = (\Delta B/B)(1-\delta)/\left[(2-\delta)\bar{\pi}+1-\delta\right] < 0$.

The number of workers (young adults) in period t+1, $N_{t+1} = \bar{N}$, remains unchanged. Equation (21) shows that the price of good y in this period, decreases by $\Delta w_{t+1}/\bar{w} = (\Delta s_t/\bar{s})/(1+\eta) = (\Delta B/B)(1-\delta)/\left[(2-\delta)\bar{\pi}+1-\delta\right]/(1+\eta) < 0$. Equation (24) shows that the fertility rate of the young adults of period t increases by

$$\Delta n_{t+1}/\bar{n} = \Delta b_t/\bar{b}' - \Delta w_{t+1}/\bar{w} = -(\Delta B/B)[\bar{\pi} + (1-\delta)/(1+\eta)]/[(2-\delta)\bar{\pi} + 1 - \delta](2B\bar{b}'+1) > 0.$$

Finally, the number of children in period t+1 and of young adults in period t+2 decreases by

$$\Delta N_{ch,t+1}/\bar{N}_{ch} = \Delta N_{t+2}/\bar{N} = \Delta\pi_t/\bar{\pi} + \Delta n_{t+1}/\bar{n} =$$
$$(\Delta B/B)[1-1/(1+\eta)](1-\delta)/[(2-\delta)\bar{\pi} + 1 - \delta] < 0.$$

b) In period t+1, we deduce from equation (22)

$\alpha\Delta z_{t+1} = \Delta w_{t+1} - (1-\alpha)\Delta s_t = [1-(1-\alpha)(1+\eta)\bar{s}/\bar{w}]\Delta w_{t+1}$. As $\bar{s} = \bar{y}\bar{w}$, we have

$\alpha\Delta z_{t+1} = [1-(1-\alpha)(1+\eta)\bar{y}]\Delta w_{t+1}$. As $(1+\eta)\bar{y} = (1-\alpha\bar{y})/(1-\alpha)$, we have $\Delta z_{t+1} = \bar{y}\Delta w_{t+1} < 0$.

We deduce from equation (23)

$$\Delta\pi_{t+1}/\bar{\pi} = (1-\delta)\Delta b_{t+1}/\bar{b}' = (\Delta B/B)(1-\delta)^2\bar{\pi}/(1+\eta)/[(2-\delta)\bar{\pi} + 1 - \delta]^2 < 0.$$

We deduce from equation (6)

$$\Delta s_{t+1}/\bar{s} = (\Delta z_{t+1} - \Delta b_{t+1})/\bar{s} = (\Delta w_{+1}/\bar{w})(2-\delta)\bar{\pi}/[(2-\delta)\bar{\pi} + 1 - \delta] < 0.$$

We deduce from equation (21)

$$(1+\eta)\frac{\Delta w_{t+2}}{\bar{w}} = \frac{\Delta s_{t+1}}{\bar{s}} - \frac{1}{\bar{y}}\frac{\Delta N_{t+2}}{\bar{N}} = \frac{1}{1+\eta}\frac{1-\delta}{(2-\delta)\bar{\pi} + 1 - \delta}\left[\frac{(2-\delta)\bar{\pi}}{(2-\delta)\bar{\pi} + 1 - \delta} - \frac{\eta}{\bar{y}}\right]\frac{\Delta B}{B}.$$

Finally, equation (24) gives

$$\frac{\Delta n_{t+2}}{\bar{n}} = \Delta b_{t+1}/\bar{b}' - \Delta w_{t+2}/\bar{w} = \frac{1}{(1+\eta)^2}\frac{1-\delta}{(2-\delta)\bar{\pi} + 1 - \delta}\left[\frac{\eta}{\bar{y}} - \frac{(1-\eta-\delta)\bar{\pi}}{(2-\delta)\bar{\pi} + 1 - \delta} - \right]\frac{\Delta B}{B}. \square$$