Intergenerational transfers and the stability of public debt when Governments understate the future

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Abstract. This paper investigates time consistent policies and reforms of intergenerational transfers. We assume that Governments have preferences, which give much weight to the living generations, and that they cannot commit themselves to future taxes and transfers, which will be decided by future Governments with different objectives. This paper shows that the economy can follow any of two equilibrium paths with perfect foresight. In the first of them, in spite of their bias, Governments will not finance the costs of their transfers to the living by increasing public debt recklessly, and by driving future generations into a process of immiserisation. They will rather put the economy on a path of equalitarian consumption flows of the successive generations, with a constant ratio of public debt to national income. In the second equilibrium, Governments will borrow more. Consumers will have to pay more and more taxes to finance the increasing cost of public debt, and they will become poorer and poorer. The mechanisms, which put an economy on one or the other equilibrium paths, are unconnected to the fundamentals of the model.

Keywords: Intergenerational transfers, Markov perfect equilibrium, overlapping generation model, time consistent policies.

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Introduction

In his excellent survey of retirement Cairncross (2004) writes: “A larger generation of old folk than ever before will need support for longer than ever before from a population of working age that is shrinking continuously in absolute size for the first time since the Black Death. Moreover, if things look bad in America and worse in continental Europe, they will one day look calamitous in some parts of the developing world”. There are two causes of this demographic transition: a widespread fall in the fertility rate and an increase in life expectancy. This demographic evolution creates a financing problem for the pay-as-you-go systems existing in many industrialised countries. To maintain the level of pensions unchanged, contributions to pension funds must be repeatedly increased. Moreover, the gap between the market interest rate and the implicit rate of return of the system becomes wider. This causes a decline in the economic condition of the young. The most natural reform to reduce the burden on the young is to decrease the level of pensions, which will lower the increase in contributions and free some income of the young generation. This can be invested in the financial markets at better rates. This policy will increase the welfare of the young and unborn, but it will decrease the income of the pensioners and the expected income of the people who are planning to retire early.

We can think of adding an ingredient to this reform. The loss of income of pensioners can be compensated by a transfer from the Government. This transfer can be financed by public borrowing. Economists, who advocate this idea, argue that the implicit public debt will just be made more visible, and that the increase in public debt will be financed by the income of the young, freed by the decrease or the slower increase of their contribution. However, the consequence of this policy will be to increase the taxation of future generations. So, it will transfer a sacrifice from the elderly currently alive to people who are not yet born. This proposal is often canvassed in current political discussions. For instance, we can refer to a very good article in *The Economist* (“From slogan to legacy”, 13 November 2004), on the state of the debate inside the American administration and the Republican party or to the Economic Focus published by this magazine on 11 December 2004. Feldstein (2005) also gives a stimulating discussion of these issues.

A quotation by Miles and Cerny (2001) will help to understand why a Government could be forced to borrow as an element of its intergenerational transfer policy: « The result that a large proportion of those alive now would be worse off if the unfunded state scheme is phased out – even though every future generation is better off – illustrates the nature of the transition problem rather clearly. Democratically elected governments facing voters who focus on the direct implications to them (and not to all future generations) of changes to state pension systems would find it hard to get support for this kind of transition plan. Table 2 suggests that once a transition from an unfunded to a funded scheme is complete welfare for all subsequent generations will be higher, but without relying on deficit financing the transition will cause certain generations to be worse off, and those generations could form a majority of voters thus permanently blocking any change."

This paper is based on the following hypothesis. Governments can organise any intergenerational transfer between living and still unborn people, under the only constraint of its intertertemporal solvency: we will assume that Governments always face their obligations to their creditors. A Government gives more weight to the living than to the unborn. Moreover, if it can set transfers and taxes for the current period, it cannot commit itself to future taxes and transfers, which will be decided by the following Governments with different objectives. However, the current Government can pressure the next Government by setting the level of public debt, which will be transmitted to its successor.

Then, a country is facing a double threat. First, a Government of a period can borrow too much and transfer an exaggerate share of the cost of its transfers to future generations. Secondly, in times of demographic transition, when financing the costs of pensions (and the health care of the elderly) becomes more difficult, a Government can increase its borrowing still more, and initiate a process of immiserisation of the future generations. Heller (2003) gives a fascinating account of the dangers for a society, which puts off costs that should be paid in the current time and which increases the burden of future generations.

The results proven in this paper only partly confirm these intuitions. We find that the economy can follow any of two equilibrium paths. In one of them, Governments borrow more and more, to improve the fate of the living. Then, the increasing cost of public debt will drive taxes to always higher levels, and the successive generations will become poorer and poorer. However, there exists another equilibrium. Then, Governments, which cannot commit to future decisions and which are especially sensitive to the fate of the living, will actually put the economy on a path of equalitarian consumption flows of the successive generations. The explanation of this result is that if a Government borrowed too much, the following Government would punish the generations, which were already alive under the first Government, by increasing their taxes. The less the first Government cared for these generations, the harsher this punishment will be. Thus, the expectation of this punishment will discipline the first Government and discourage it of borrowing too much. The paper will also show that, for the last equilibrium, in time of demographic transition, the sacrifice, which will have to be made, will be shared in an equalitarian way across generations.

A difference between the first and the second equilibrium is the extent of the punishment by a Government of the generation, which was already alive under the previous Government, the one which could borrow too much. This punishment is higher for the second than for the first equilibrium. However, in both situations, expectations are perfect. The mechanisms, which put an economy on one or the other equilibrium paths, are unconnected to the fundamentals of our model. Thus, we are left with the open question of which sunspots will decide if the future generations will enter or not an immiserisation process. The fact that these sunspots cannot be directly connected to demographic transition, does not prevent the possibility that we would enter a period when some Western economies switch from a path of stable public debt to a path of increasing Government indebtedness.

The analysis of intergenerational transfers is usually made by using an overlapping generation model. However, even the simplest versions of this model are complex under the assumption of a closed economy (see for instance Azariadis (1993) or de la Croix and Michel (2002)). The main reason for this complexity comes from the feedback from saving to investment. Hence, in this paper we will use an overlapping generation model of a small open economy. In this case Fisher's separation theorem separates saving from investment decisions as agents can borrow or lend at a fixed interest rate. Then, we will be able to solve the model fully and to answer our questions in an analytical manner. Successive Governments and generations of consumers will participate to a dynamic game, where Governments will adopt Markov strategies, and where the private sector will adapt its decentralised decisions to the current and expected decisions of Governments.

The first section will present the model. The second section will describe the dynamic game that is played between successive Governments and generations of people. It will also explain how Governments determine their public transfers and debts. The third section will give a series of lemma characterising the equilibriums of the model. The fourth section will investigate the first equilibrium, where the consumption and the welfare of successive generations stay constant over time. The fifth section will investigate the second equilibrium, where successive generations enter an immesirising process.

1. An overlapping generations model

The model is constructed for a small open economy. Domestic and foreign goods are perfect substitute and the international capital market is perfect. Time is assumed to be discrete, 0 denotes the current period, and agents hold perfect expectations. All economic variables are expressed in real terms. $N(t)$ consumers are born at the beginning of period t. They will die at the end of period $t+1$. They work for the first period of their life. Then they retire. Each worker receives wages $w(t)$. The number of births increases over time at rate $1+n>0$. Thus, we have $N(t)=N(1+n)^{t+1}$, for $t\geq -1$, where $N=N(-1)$ denotes the number of people entering the second period of their life at the beginning of period 0. We will norm the population by setting $N=1$. The domestic interest rate is equal to the interest rate in the rest of the world, $i^{*}>0$, which is exogenous and constant over time. We will need the following assumption:

Assumption 1. The interest rate in the rest of the world is higher than the growth rate of population: $i^* > n$.

1.1. Firms

Employment in period t equals $N(t)$. Domestic output in this period $Y(t)$ is given by a Cobb-Douglas function

(1) $Y(t) = PK(t)^{\alpha} N(t)^{1-\alpha}$

To simplify the model, we will assume that the global productivity of factors P stays constant over time. Thus, the natural growth rate of the economy is equal to n that is the expansion rate of the active population. $K(t)$ denotes the amount of capital used in period t. We will assume that capital does not depreciate. We have the two following marginal conditions

(2) $w(t) = (1-\alpha)Y(t)/N(t)$

(3) $i^* = \alpha Y(t)/K(t)$

Equations (1), (2) and (3) give the expression of the wage rate, domestic output and capital

(4) $w(t) = w = (1-\alpha)P^{1/(1-\alpha)}(\alpha / i^*)^{\alpha/(1-\alpha)}$

(5), $Y(t) = P^{1/(1-\alpha)}\left(\alpha / i^*\right)^{\alpha/(1-\alpha)}(1+n)^{t+1}$

(6), $K(t) = \left(\alpha P / i^*\right)^{1/(1-\alpha)}(1+n)^{t+1}$

1.2. The public sector

$s_1(t)$ and $s_2(t)$ denote net public transfers made by the Government, respectively to each young and to each old person, in period t. To simplify the model we will assume that the Government does not consume goods. Public debt at the end of period t, deflated by the active population living in the period, will be denoted as $B(t)$. The budgetary equilibrium of the Government is

(7) $(1+n)B(t) = (1+n)s_1(t) + s_2(t) + (1+i^*)B(t-1)$

This equation determines the dynamics of total public debt $(1+n)^{t+1}B(t)$ (the initial level of total public debt $B(-1)$ is given). We will assume that the sequence of net public transfers is consistent with a bounded public debt per worker $B(t)$. Under Assumption 1, this condition implies the intertemporal solvency of the Government: $B(t)\left[(1+n)/(1+i*)\right]^t \underset{t\to\infty}{\to} 0$.

1.3. Plan of a consumer born at the beginning of period $t \geq 0$

We will assume that consumers receive no endowment at their birth and leave no bequest when they die. The non capital incomes of a consumer born at the beginning of period t, in the two periods of its life, respectively are: $w + s_1(t)$, and: $s_2(t+1)$. The wealth at birth of this consumer is

(8) $W(t) \equiv w + s_1(t) + s_2(t+1)/(1+i^*)$

This agent consumes $C_1(t)$ and $C_2(t+1)$ in the two periods of his life. These consumption flows must satisfy the budgetary constraint

(9) $C_1(t) + C_2(t+1)/(1+i^*) = W(t)$

The wealth of this consumer at the end of period t, which is identical to national private wealth per worker, is

(10) $A(t) = w + s_1(t) - C_1(t)$

We assume that consumers take their lifetime decisions at the time of their birth. They perfectly forecast their future incomes. Their discount rate is $\beta > 0$ and their preference for consumption is logarithmic. Thus, the utility at birth of a consumer born at the beginning of period t is

(11) $U(t) = \ln\left[C_1(t)\right] + \ln\left[C_2(t+1))\right]/(1+\beta)$

We could have assumed an incompressible level of consumption per period: $\overline{C} > 0$. Thus, the utility of consuming C would have been: $\ln(C - \overline{C})$, instead of: $\ln(C)$. Nothing would have to be changed in the paper, except that consumption would represent the difference between actual consumption and its incompressible level, and wages would represent the difference between actual wages and the cost of the incompressible consumption over lifetime: $\overline{C}\left[1+1/(1+i^*)\right]$. We also remade all the calculations of the paper for the general class of CRRA functions: $C^{1-\sigma}/(1-\sigma)$, with $\sigma > 0$. The case of logarithmic preferences investigated in the paper corresponds to the case when σ is equal to 1. The mathematical expressions we got are a little more complicated than the ones given in the paper, but the qualitative results are unchanged. Thus, we can think that these results, especially the coexistence of two equilibrium paths for the economy, are robust.

The maximisation of the utility function (11) under the budgetary constraint (9) gives the consumption of this cohort in the two periods of its life

(12) $C_1(t) = W(t)(1+\beta)/(2+\beta)$

(13) $C_2(t+1) = W(t)(1+i^*)/(2+\beta)$

1.4. Plan of a consumers born at the beginning of period -1

This consumer enters the second part of his life at the beginning of period 0. Then, his wealth inherited from the past is $A(-1)$. Moreover, this consumer receives public transfers $s_2(0)$. His consumption is equal to the sum of his wealth, the interest income earned on this wealth and his non-wealth income

(14) $C_2(0) = (1+i^*)A(-1) + s_2(0)$

The utility of this consumer at the beginning of period 0 is

(15) $U(-1) = \ln\left[C_2(0)\right]$

We make the following assumption, which implies that the total wealth at birth of each consumer is positive.

Assumption 2. The Government must satisfy the constraints: $s_2(0) + (1+i^*)A(-1) > 0$, *and:* $w + s_1(t) + s_2(t+1)/(1+i^*) > 0$, *for* $t \geq 0$.

We will not write the last equations of the model, which determine the surplus of the trade balance, the stock of foreign assets held by the country, etc. These equations do not interact with the part of the model we have presented and we are not interested by the dynamic paths followed by these variables.

2. How does the Government determine public transfers and debt?

We can easily prove that the sum of the discounted wealth at birth of all generations, computed in period 0, is independent of intergenerational transfers. More precisely we have the relation

$$(16)\quad \sum_{t=0}^{\infty}\left[C_1(t)+C_2(t+1)/(1+i^*)\right]\left[(1+n)/(1+i^*)\right]^{t+1}+C_2(0)/(1+i^*)=\\ (1+n)w/(i^*-n)+A(-1)-B(-1)$$

This result shows that a reform of the public transfer policy to the successive generations will improve the welfare of some generations and lead to a decline in the utility of other generations. This efficiency result is well known (see for instance Azariadis, 1993, or Feldstein and Liebman, 2002). Its consequence is that the system of intergenerational transfers, which prevails in an economy, results from Government's arbitrage between the various generations.

We will introduce a social welfare function of the Government at time 0, with the following specification

$$(17)\quad \Omega(0)=U(-1)+A(1+n)U(0)\,,\ A>0$$

This function gives different weights to the utilities $U(-1)$ of the elderly alive in period 0 and $(1+n)U(0)$ of the cohort of the youth who are also currently alive. The Government does not care for generations, which will be born in later periods. Other social welfare functions could be considered. However, this specification is simple and will be sufficient to establish our results.

At time 0, the Government can set only its decisions for this period and cannot commit itself to decisions, which will be implemented by future Governments. Thus, when the Government of period 0 makes its choice it must anticipate the reactions of the Governments of the future, the social welfare functions of which will be: $\Omega(t) = (1+n)^t \left[U(t-1) + A(1+n)U(t)\right]$, $t \geq 1$. In this expression, $U(t-1)$ and $U(t)$ respectively represent the utility of each old person and of each young person, alive in period t, computed for the whole rest of their life. We notice that the relative weight of the various cohorts in the social welfare function changes over time. Therefore, the solution to the problem will exhibit dynamic inconsistency: the actions for the future periods that the Government would chose in period 0, if it were able to pre-commit, differ from those it will find optimal when in periods 1, 2, etc.

The standard approach to analyse sequential decisions problems with time-inconsistent preferences is to view the decision-maker in each period t as a distinct player, in the sense of non-cooperative game theory (see for instance Vieille and Weibull (2005)). As a result, one obtains a sequential game with infinitely many players, who each acts only once, but who will care not only about the material payoff in their own period but also about the payoffs in subsequent period. By a solution is usually meant a subgame perfect equilibrium in the so defined game, that is, a strategy profile that induces a Nash equilibrium in every subgame. Each player then maximises the future utility streams as evaluated from (and including) the current period, and given the strategies of all future players.

A pure strategy $\sigma(t)$ of the Government of period t is a map from the history up to period t to the set of actions. A pure strategy profile $\sigma = (\sigma(t))_{t \in N}$ is a sequence of pure strategies for $t \geq 0$. A strategy profile is subgame perfect if for each $t \in N$ and each possible history h before period t, the decision $\sigma(t;h)$ of the Government of period t is optimal, given this history and the decision rules used by future Governments of periods $t+1$, $t+2$, etc.

The problem with subgame perfect equilibriums is that they often give a large multiplicity of solutions with distinct payoffs. Thus, we will use a refinement of this concept, the Markov perfect equilibrium, which is well adapted to our problem. Maskin and Tirole (2001) give the following definition.

"Consider a dynamic game in which, in every period t, player i's payoff π_t^i depends only on the vector of players'actions, a_t, that period, and on the current (payoff-relevant) "state of the system" $\theta_t \in \Theta_t$. That is, $\pi_t^i = g_t^i(a_t, \theta_t)$. Suppose, furthermore, that player i's possible actions A_t^i depend only on θ_t: $A_t^i = A_t^i(\theta_t)$ and that θ_t is determined by the previous actions a_{t-1} and state θ_{t-1}. Finally, assume that each player maximizes a discounted sum of per period payoffs: $\sum_t \delta^{t-1}\pi_t^i$. In period t, the history of the game, h_t, is the sequence of previous actions and states $h_t = ((a_1, \theta_2),...,(a_{t-1}, \theta_t))$. But the only aspect of history that directly affects player i's payoffs and action sets starting in period t is the state θ_t. Hence, a Markov strategy in this model should make player i's period t action dependent only on the state θ_t rather than on the whole history h_t… We shall define a Markov Perfect Equilibrium to be a subgame perfect equilibrium in which all players use Markov strategies". These authors add: "Many economic models entail games that are stationary in the sense that "they look the same" starting in any period", i.e., they do not depend on calendar time (clearly, such games must have an infinite horizon). For these games it is natural to make Markov strategies independent of calendar time as well".

The dynamic game of our model can be described in the following way.

The initial state in period $t \geq 0$ is defined by the amounts of public debt $B(t-1)$ and of national private wealth $A(t-1)$, deflated by the number of living old persons. The Government of this period sets the transfers to the living youth and elderly, $s_1(t)$ and $s_2(t)$. Then, these people determine their consumption. The behaviour of the elderly is passive: each of them consumes his total wealth and income: $C_2(t) = (1+i^*)A(t-1) + s_2(t)$. Each young person determines his consumption of the period and his expected consumption of next period, $C_1(t)$ and $C_2(t+1)$, according to equations (12) and (13). He needs to forecast the transfer he will receive from the Government of period $t+1$,

$s_2(t+1)$. However, although he knows that this transfer will depend on the aggregated choices of all the young consumers, he also knows that he is too small for his actions to have any effect on these aggregated choices. So, he will consider his expectation of $s_2(t+1)$, as a value and not as a function.

The Government of period t perfectly forecasts the reaction to its actions by the consumers of the same period. Moreover, it knows that the transfers decided by the next Government will be a function of the state of the economy at the beginning of period $t+1$: $s_2(t+1) = f(B(t), A(t))$. The Government of period t will take its decision by maximising its social welfare function under the constraints given in Assumption 2.

The definition of Markov strategies does not put any constraint on the shape of the reaction function f. We will assume here that this function is linear, and will discuss this assumption in Lemma 4

(18) $s_2(t+1) = s_2^{'} - a_1 B(t) - a_2 A(t)$, for $t \geq 0$.

We deduce, from the behaviour of the youth of period t and from equation (10)

(19) $(2+\beta)A(t) - w = s_1(t) - \dfrac{1+\beta}{1+i*} s_2(t+1)$

Then, under the condition $a_2 \neq (1+i*)\dfrac{2+\beta}{1+\beta}$, the reaction of the Government of period $t+1$ expected by the Government of period t can be rewritten:

(20) $s_2(t+1) = s_2 - \lambda B(t) - \mu s_1(t)$, with: $s_2 = \dfrac{(2+\beta)s_2^{'} - a_2 w}{2+\beta - a_2(1+\beta)/(1+i*)}$,

$\mu = \dfrac{a_2}{2+\beta - a_2(1+\beta)/(1+i*)}$, $\lambda = \dfrac{a_1(2+\beta)}{2+\beta - a_2(1+\beta)/(1+i*)}$.

In the rest of the paper we will use the constant parameters s_2, λ and μ, instead of the original parameters $s_2^{'}$, a_1 and a_2.

3. The characterisation of the equilibriums of the model

The current period is $t=0$. We will compute the decisions executed in this period by the current Government and the decisions, which are expected to be executed in future periods by future Governments. We will see that these decisions are consistent with the assumed reaction function. The Government of period t only cares for the living youth and elderly and its transfers can only increase the satisfaction of these agents. This Government can increase these transfers by borrowing more. However, in this case, the Governments of period $t+1$ will react by punishing the living youth of period t, by taxing them in proportion of public debt when they grow old. Moreover, the transfers, which will be given to the elderly by the Government of period $t+1$, will depend on the transfers these people received from the previous Government when they were young.

We can easily notice that the program of the Government of period t has no solution if $1+i^*-\mu-\lambda \neq 0$ (see the proof of Lemma 1). The reason is that under this assumption, this Government could control the consumptions in both periods of the youth of period t, and of course the consumption of the elderly, and that because the reaction of the Government of period $t+1$ would be inadequate. Thus, the Government of period t would have no reason for not setting these three consumptions to infinity. We will assume that the reaction function of the Government of period $t+1$ forbids this behaviour by the Government of period t:

Assumption 3. The reaction function of Governments must satisfy the constraint: $1+i^*-\mu-\lambda=0$.

We will start by establishing the following lemma.

Lemma 1. The policy, which is implemented by the Government of period $t \geq 0$ *is determined by the equation*

$$(21)\quad \frac{1}{C_2(t)} = \frac{2+\beta}{1+i^*}\frac{\lambda A}{1+\beta}\frac{1}{W(t)}$$

Proof. See Appendix.

The decisions of the Government of period t and the reaction function of the Government of period $t+1$ appear implicitly in the expressions of $C_2(t)$ and $W(t)$. We can combine this equation and the accounting identities giving the dynamics of public debt $B(t)$ and of the wealth of the elderly $A(t)$, to determine the dynamics of the economy. We get the following lemma.

Lemma 2. The sequence of transfer policies is determined by the reaction function

(22) $s_2(t+1) = s_2 - \lambda B(t) - (1+i^* - \lambda)s_1(t)$

equations (7) and (19) and the following equation

(23) $C_2(t+1) = \left[s_2(t+1) + (1+i^*)A(t)\right] = \left[s_2(t) + (1+i^*)A(t-1)\right]\lambda A/(1+\beta) = C_2(t)\lambda A/(1+\beta)$

for $t \geq 0$, *with* $A(-1)$ *and* $B(-1)$ *given.*

Proof. See Appendix.

Lemma 3 will establish a condition, which must be satisfied by the reaction function of the Governments:

Lemma 3. The existence of an equilibrium requires that either: $\lambda A = 1+\beta$, *or:* $\lambda = \dfrac{1+i^*}{1+n}\dfrac{1}{A/(1+\beta)+1/(1+n)}$.

Proof. See Appendix.

We have assumed that the reaction function of Governments is linear. The following lemma will show that this assumption is weaker than it looks.

Lemma 4. The reaction function of the Government of period $t+1$*:* $s_2(t+1) = f\left[B(t), s_1(t)\right]$ *cannot simultaneously satisfy the following conditions*

a) The function f *is continuously differentiable.*

b) The function

$$g\left[s_2(t)+(1+i^*)B(t-1)\right]=\arg\underset{s_1(t)}{Max}\left\{s_1(t)+f\left\{s_1(t)+\left[s_2(t)+(1+i^*)B(t-1)\right]/(1+n),s_1(t)\right\}/(1+i^*)\right\}$$

is univoque and continuously differentiable (so f cannot be linear).

However, this lemma is insufficient to show that the linear reaction function that we have assumed in this paper is the only possible reaction function. There is still the possibility that there exist other reaction functions, which would for instance be discontinuous. Even if we exclude this possibility, Lemma 3 suggests that the model has two equilibriums. We will show that such is the case and we will investigate them. But, before, we need another assumption, which will imply Assumption 2.

Assumption 4. The initial conditions of the economy satisfy: $(1+n)w+(i^*-n)\left[A(-1)-B(-1)\right]>0$.

4. An equalitarian equilibrium. The case when: $\lambda=(1+\beta)/A$

Proposition 1 will give all the features of this equilibrium.

Proposition 1. The first equilibrium path of the economy is described by the following

a) The transfers to the elderly in period 0 are given by

(24) $$s_2(0)=\frac{(1+i^*)(1+n)}{(1+n)(2+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\}-(1+i^*)B(-1)$$

b) The constant part of the transfers to the elderly in the following periods is given by

(25) $$s_2=-\frac{(i^*-n-\lambda)(1+i^*)}{(1+n)(2+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\}$$

c) The consumption flows of each old and young persons are constant over time and given by

(26) $$C_2(t)=\frac{1+i^*}{(1+n)(2+\beta)+(i^*-n)}\left\{(1+n)w+(i^*-n)\left[A(-1)-B(-1)\right]\right\},$$

(27) $$C_1(t)=\frac{1+\beta}{(1+n)(2+\beta)+(i^*-n)}\left\{(1+n)w+(i^*-n)\left[A(-1)-B(-1)\right]\right\}$$

d) The transfers to the youth in period $t\geq 0$, $s_1(t)$, can be set to arbitrary levels.

e) Public indebtedness per worker, is given by

$$(28)\ B(t) = s_1(t) + \frac{1+i^*}{(1+n)(2+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\},\ \textit{for}\ t \geq 0$$

f) Net national wealth per worker is constant over time

$$(29)\ A(t)-B(t) = A(-1)-B(-1),\ \textit{for}\ t \geq 0$$

g) The transfers to the elderly after period 0 are given by

$$(30)\ s_2(t+1) = -(1+i^*)s_1(t) - \frac{(i^*-n)(1+i^*)}{(1+n)(2+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\},\ t \geq 0$$

Proof. See Appendix.

To understand the economic meaning of Proposition 1, we will introduce the concept of admissible path of the consumption flows of the successive generations. First, such paths must satisfy the intertemporal budget constraint (16). Moreover, they must be consistent with the ratio, set by each consumer between its consumption in the two parts of its life, and given by equations (12) and (13)

$$(31)\ C_2(t+1)/C_1(t) = (1+i^*)/(1+\beta),\ \text{for}\ t \geq 0$$

If we substitute this expression in equation (16), the admissible path of the consumption of the elderly is given by

$$(32)\ \frac{2+\beta}{1+i^*}\sum_{t=0}^{\infty} C_2(t+1)\left[(1+n)/(1+i^*)\right]^{t+1} + C_2(0)/(1+i^*) = (1+n)w/(i^*-n) + A(-1) - B(-1)$$

The steady state solution of this equation, $C_2(t) = C_2$ for $t \geq 0$, is identical to equation (26).

The utility of the old generation living in period 0 is given by equation (15): $U(-1) = \ln\left[C_2(0)\right]$. The utility of the generation born in period $t \geq 0$ can easily be deduced from equation (11)

$$(33)\ (1+n)^{t+1}U(t) = (1+n)^{t+1}\frac{2+\beta}{1+\beta}\ln\left[C_2(t+1)\right]$$

Let us assume for a moment that the Government of period 0 could commit itself to implement all its decisions. Moreover, let us assume that this Government cares for all generations and uses the following social welfare function

(34) $U(-1)+D\sum_{t=0}^{\infty}\left[E(1+n)\right]^{t+1}U(t)$, with $D>0$ and $0<E<1/(1+n)$.

This Government will select the optimal admissible path of the consumption of the elderly by maximising (34) under the constraint (32). The first order conditions of this program is

(35) $C_2(t+1)=\left[D/(1+\beta)\right]\left[(1+i^*)E\right]^{t+1}C_2(0)$

We will get the same result as equation (26) if and only if

(36) $D=1+\beta$, and: $E=1/(1+i^*)$

Then, the social welfare function of the Government becomes $U(-1)+\frac{1+\beta}{1+i^*}\sum_{t=0}^{\infty}\frac{(1+n)^{t+1}U(t)}{(1+i^*)^t}$

We can sum up the above considerations in the following proposition.

Proposition 2. The first equilibrium path of the model is the only admissible steady state path consistent with the initial conditions. This path would be selected by a Government who could commit itself to the execution of its decisions in the future if its social welfare function satisfies the constraints (36).

Actually, we have assumed in this paper that a Government cannot commit itself to decisions, which will be implemented by future Governments, and that it only cares for the living generations. However, in spite of this selfishness (or short-sightedness), the decisions taken by the successive Governments, will result in a consumption flow per head, which does not change over time. This equalitarian allocation of consumption between generations implies that the Government of a period does not transfer the cost of its generosity with the living to the unborn, and does not deteriorate the fate of these last generations. We can notice that the first equilibrium path of the model is independent of the weight A that the Government gives to the youth in comparison to the elderly.

The first solution of the model has the property of hysteresis: the net national wealth per worker in each period and the consumption of each person are constant over time and permanently depend on the initial values of the net national wealth (see Laffargue (2004)). Changes in the transfers to the youth $s_1(t)$ for $t \geq 0$, have no effect on the consumptions and the welfare of the agents, nor on the net national wealth. However, these changes affect public debt $B(t)$ and national private wealth $A(t)$. If the Government of period t wants to increase the wealth of the young generation to a level higher than the one that the Government of period $t+1$ will consider as optimal, and increases its transfers $s_1(t)$, the latter Government will destroy this attempt by reducing its transfers to the same cohort $s_2(t+1)$ (see equation (30)). This lack of effects of transfers to the young generation is a kind of Ricardian equivalence, which is valid in this overlapping generation model with endogenous public transfers policy.

The Government of period t can set $s_2(t)$ and the welfare of the old generation in period t to the level it wants because no future Government can correct this decision. However, the Government of period $t+1$ can punish the Government of period t by increasing its taxes on the generation, which was young in period t. The parameter λ measures the extent of this punishment, which is inversely proportional to the weight of the youth, A, in the social welfare function of the government of period t. The more this Government cares for this generation, the less severe will have to be the punishment necessary to forbid this Government of borrowing too much.

Before making our next comment on the economic meaning of the first equilibrium of the model, we need a last series of assumptions.

Assumption 5. The initial conditions of the model and the transfers to the youth satisfy the following constraints: $w-(2+\beta)\left[A(-1)-B(-1)\right]>0$, *and:*

$$s_1(t) < -\frac{(i^*-n)/(1+i^*)}{(1+n)(1+\beta)+(i^*-n)}\left\{w-(2+\beta)\left[A(-1)-B(-1)\right]\right\}.$$

Under these assumptions, we deduce from equation (30) that the transfers to the elderly satisfy the following constraint

(37) $0 < s_2(t+1) < -(1+i^*)s_1(t)$, with $t \geq 0$

Thus, we are in the situation where each young person pays a positive amount of taxes ($-s_1(t)$), and each old person receives a positive amount of transfers ($s_2(t+1)$). However, the return on taxes that is the ratio between what a person receives when he is old to what he paid when he was young, less 1, is less than the interest rate. However, when the rate of growth of population tends to the interest rate, the return on taxes converges to the interest rate.

Finally, we can investigate how the first equilibrium reacts to demographic transition. This will be defined as a decrease in the growth rate n of population. More precisely, we will assume that n is constant and relatively large until period $T-1>0$, then it is set to a lower value at and after period T. Equation (29) shows that the net national wealth per worker, $A(t)-B(t)$, does not change at and after T. We can deduce from equations (26) and (27) and from Assumption 5 that, at and after T, consumptions per head, $C_1(t)$ and $C_2(t)$, decrease by the same proportion.

We deduce from equation (30) that $s_2(t+1)+(1+i^*)s_1(t)$ decreases, that is the return on taxes decreases as a consequence of the demographic transition. Equation (24) shows that in period T, the transfer to each old person, $s_2(T)$, decreases. Finally, if the taxes paid by the youth, $-s_1(t)$ are kept unchanged after the demographic transition, public debt and private wealth per worker, $B(t)$ and $A(t)$, decrease. The transfers to each old person after period T, $s_2(t+1)$, also decrease.

An interesting result is that the Government of period T will not try to prevent the decrease in the consumption of its living elderly, by increasing its transfers to them and by financing this cost by public borrowing, even if the weight of the living youth in its preference is low. The reason is that if this Government increased its debt, the following Government would punish the youth of period T, and this punishment would be the more severe the less the Government of period T would care about this youth.

5. An equilibrium immesirising future generations. The case when : $\lambda = \frac{1+i^*}{1+n}\frac{1}{A/(1+\beta)+1/(1+n)}$

Lemma 2 showed that the expansion rate of consumption per head is equal to $\lambda A/(1+\beta)$. The following assumption will imply that this rate is smaller than 1, or more precisely: $i^* - n < \lambda < (1+\beta)/A$. Thus, the people of the successive generations will consume less and less.

Assumption 6. The weight given to the young in the social welfare function of Governments satisfy the constraint: $A < (1+\beta)/(i^* - n)$.

The developments of section 4 suggested that $A \sim DE = (1+\beta)/(1+i^*)$. If: $\beta \sim i^*$ that is if the discount rate of consumers is near the international interest rate, we have: $A \sim 1$. Thus, the constraint of assumption 6 is approximately equivalent to: $n > -1$, which is a constraint that we set at the beginning of section 1. So, this assumption is quite realistic. Proposition 3 will give all the features of the new equilibrium.

Proposition 3. The second equilibrium path of the economy is described by the following

a) The constant part of the transfers to the elderly in periods following period 0 is given by

(38) $s_2 = w[\lambda/(i^* - n) - 1](1+i^*)$

b) The transfers to the elderly in period 0 are given by

(39) $s_2(0) = \frac{1+i^*}{2+\beta+\lambda/(1+n)}\{w\lambda/(i^* - n) - (2+\beta)A(-1) - \lambda B(-1)/(1+n)\}$

c) The consumption flows of each old and young person in period $t \geq 0$ *are given by*

(40) $C_2(t) = \frac{\lambda}{i^*-n}\frac{1+i^*}{(1+n)(2+\beta)+\lambda}\{(1+n)w+(i^*-n)[A(-1)-B(-1)]\}\left(\frac{\lambda A}{1+\beta}\right)^t$,

(41) $C_1(t) = \frac{\lambda}{i^*-n}\frac{1+\beta}{(1+n)(2+\beta)+\lambda}\{(1+n)w+(i^*-n)[A(-1)-B(-1)]\}\left(\frac{\lambda A}{1+\beta}\right)^{t+1}$

d) The transfers to the youth in period $t \geq 0$, $s_1(t)$*, can be set to arbitrary levels.*

e) Public indebtedness per worker, is given by

(42) $B(t) = s_1(t) + w(1+i^*)/(i^*-n) - (2+\beta)C_2(t+1)/\lambda$, *for* $t \geq 0$

f) The transfers to the elderly after period 0 are given by

(43) $s_2(t+1) = -(1+i^*)s_1(t) - w(1+i^*) + (2+\beta)C_2(t+1)$, *for* $t \geq 0$

g) Private national wealth per worker is given by

(44) $A(t) = s_1(t) + w - (1+\beta)C_2(t+1)/(1+i^*)$, *for* $t \geq 0$

Proof. See Appendix.

This second equilibrium path of the economy is still admissible and efficient. We can easily show that the consumption in period 0 of each old person is higher than for the first equilibrium. However, the consumption of a person in each stage of his life decreases from generations to generations and converges to 0. Public indebtedness per worker $B(t)$ increases over time and converges to $s_1(t) + w(1+i^*)/(i^*-n)$. Net national wealth per worker, $A(t) - B(t)$, converges to the negative amount $-w(1+n)/(i^*-n)$. In the long run, each person will pay the total amount of his wealth at birth to the Government, to finance the stabilisation of the public debt, which results from the profligacy of previous Governments.

We still get the same Ricardian equivalence result as for the first equilibrium: the transfers to the young, $s_1(t)$, do not matter.

The differences between this equilibrium and the previous one comes from a lower rate of punishment λ. Thus, a Government is less disciplined by its successor than in the previous equilibrium and uses public borrowing more freely.

Demographic transition is a bit paradoxical for this equilibrium. First, we notice that it increases the value of the punishment parameter λ, and so it reduces the rate of immiserisation of the future generations: $1-\lambda A/(1+\beta)$. Secondly, we cannot get any result on the direction of the changes of consumption or public indebtedness per worker when demographic transition starts. However, the ratio between the consumption of a young person and an old person in the same period: $C_1(t)/C_2(t)=\lambda A/(1+i^*)$, increases.

Conclusion

This paper uses an overlapping generations model of an open economy to analyse the setting of time consistent intergenerational transfers policies. Governments make their decisions without putting weight on the welfare of future generations. They cannot commit to decisions, which will be executed by their successors. However, they can constraint the choices of the Governments of the future by setting the level of public debt (Governments always face their obligations with their creditors). However, a Government can punish a predecessor, which borrowed too much, by increasing taxes on people who were already alive under this previous Government (who so cared for the welfare of these consumers).

Our model has two perfect foresight equilibriums. In the first one, where the rate of punishment is high, consumption and public debt per head stay constant over time. Thus, future generations will not be sacrificed for the benefit of the current ones. Demographic transition does not change this result. It only makes all the consumers uniformly poorer. In the second equilibrium, where the rate of punishment is lower, the initial consumption of the elderly is higher than for the first equilibrium.

However, consumption decreases over time and converges to 0. Public debt per head increases over time and converges to a high level.

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APPENDIX. Proofs

Proof of Lemma 1

The program, which determines the choices of the Government of time $t \geq 0$ is

$$\underset{s_1(t), s_2(t)}{Max} \left[U(t-1) + A(1+n)U(t)\right]$$

$$U(t-1) = \ln\left[(1+i^*)A(t-1) + s_2(t)\right]$$

$$U(t) = \frac{2+\beta}{1+\beta} \ln\left[w + s_1(t) + s_2(t+1)/(1+i^*)\right]$$

$$s_2(t+1) = s_2 - \lambda B(t) - \mu s_1(t)$$

$$B(t) = s_1(t) + s_2(t)/(1+n) + \left[(1+i^*)/(1+n)\right]B(t-1)$$

with $A(t-1)$, $B(t-1)$, s_2, λ given and $\mu = 1 + i^* - \lambda$.

This program is equivalent to the program

$$\underset{s_1(t), s_2(t)}{Max} \left[U(t-1) + A(1+n)U(t))\right]$$

$$U(t-1) = \ln\left[(1+i^*)A(t-1) + s_2(t)\right]$$

$$U(t) = \frac{2+\beta}{1+\beta} \ln\left[w + s_2/(1+i^*) - \lambda s_2(t)/(1+n)/(1+i^*) - \lambda B(t-1)/(1+n)\right]$$

with $A(t-1)$, $B(t-1)$, s_2, λ given.

The value taken by the objective function of this program is independent of the transfers $s_1(t)$ to the youth. Its maximisation relatively to the transfers to the elderly $s_2(t)$ gives equations (21). It is easy to show that the second order condition is satisfied. □

Proof of Lemma 2

Equation (23) results from equations (13) and (21). □

Proof of Lemma 3

Equations (19) and (7) can be written

$$(2+\beta)C_2(t+1) - w(1+i^*) = s_2 + \lambda[s_1(t) - B(t)],$$

$$(1+n)[s_1(t) - B(t)] - (1+i^* - \lambda)[s_1(t-1) - B(t-1)] + s_2 = 0 \text{, for } t \geq 1.$$

We use the first of these equations to eliminate $s_1(t) - B(t)$ and $s_1(t-1) - B(t-1)$ from the second equation. We get , for $t \geq 1$

$$(i^* - n)s_2 = -(2+\beta)(1+n)\left[C_2(t+1) - \frac{1+i^*-\lambda}{1+n}C_2(t)\right] - w(1+i^*)(i^* - n - \lambda) =$$

$$-(2+\beta)(1+n)\left[\frac{\lambda A}{1+\beta} - \frac{1+i^*-\lambda}{1+n}\right][(1+i^*)A(-1) + s_2(0)]\left(\frac{\lambda A}{1+\beta}\right)^t - w(i^* - n - \lambda)(1+i^*)$$

As $i^* > n$, s_2 can be constant over time only if: $\lambda A = 1+\beta$ or if: $\lambda = \frac{1+i^*}{1+n}\frac{1}{A/(1+\beta) + 1/(1+n)}$.

□

Proof of Lemma 4

We will assume in the proof that: $n = 0$, which will simplify the notations. The social welfare function of the Government of period t is

$$\ln[(1+i^*)A(t-1) + s_2(t)] + A\frac{2+\beta}{1+\beta}\ln[w + s_1(t) + s_2(t+1)/(1+i^*)]$$

This Government considers $A(t-1)$ as given and forecasts the reaction function of the Government of period $t+1$: $s_2(t+1) = f[B(t), s_1(t)]$. We will assume that this function is continuously differentiable. We remind that

$$B(t) = s_1(t) + s_2(t) + (1+i^*)B(t-1)$$

We will assume that the function $s_1(t) + f[s_1(t) + s_2(t) + (1+i^*)B(t-1), s_1(t)]/(1+i^*)$ has a unique maximum in $s_1(t)$: $s_1(t) = g[s_2(t) + (1+i^*)B(t-1)]$, with function g continuously differentiable. We substitute this expression of $s_1(t)$ in the above function and we get

$$s_1(t) + f[s_1(t) + s_2(t) + (1+i^*)B(t-1), s_1(t)]/(1+i^*) = h[s_2(t) + (1+i^*)B(t-1)]$$

The function h is continuously differentiable.

We maximise the social welfare function of the Government relatively to $s_2(t)$ and we get the necessary first-order condition

$$\frac{1}{(1+i^*)A(t-1) + s_2(t)} + A\frac{2+\beta}{1+\beta}\frac{h'[s_2(t) + (1+i^*)B(t-1)]}{w + h[s_2(t) + (1+i^*)B(t-1)]} = 0$$

We deduce from equation (19)

$$(1+i^*)A(t-1) = \frac{1+i^*}{2+\beta}[w + s_1(t-1)] - \frac{1+\beta}{2+\beta}s_2(t)$$

Then, the first-order condition becomes

$$\frac{1}{(1+i^*)[w + s_1(t-1)] + s_2(t)} + \frac{A}{1+\beta}\frac{h'[s_2(t) + (1+i^*)B(t-1)]}{w + h[s_2(t) + (1+i^*)B(t-1)]} = 0$$

For $t \geq 1$ we have

$$\begin{aligned} s_2(t) &= h[s_2(t-1) + (1+i^*)B(t-2)] - g[s_2(t-1) + (1+i^*)B(t-2)] \\ &= h[B(t-1) - s_1(t-1)] - g[B(t-1) - s_1(t-1)] \end{aligned}$$

Let us set: $x = B(t-1) - s_1(t-1)$, and $y = s_1(t-1)$. We have the functional equation

$$\frac{1}{(1+i^*)(w+y) + h(x) - g(x)} = -\frac{A}{1+\beta}\frac{h'[h(x) - g(x) + (1+i^*)(x+y)]}{w + h[h(x) - g(x) + (1+i^*)(x+y)]}$$

or

$$\begin{aligned} &(1+i^*)(w-x) = F[h(x) - g(x) + (1+i^*)(x+y)] \equiv \\ &-\frac{1+\beta}{A}\frac{w + h[h(x) - g(x) + (1+i^*)(x+y)]}{h'[h(x) - g(x) + (1+i^*)(x+y)]} - \{h(x) - g(x) + (1+i^*)(x+y)\} \end{aligned}$$

As the right-hand side must be independent of y, the function F must be constant. However, in this case it cannot depend linearly on x. □

Proof of Proposition 1

a) The last equation of the proof of Lemma 3 gives

$$(i^*-n)s_2 = (i^*-n-\lambda)\{(2+\beta)[(1+i^*)A(-1)+s_2(0)]-w(1+i^*)\}$$

If we notice that: $C_2(0)=C_2(1)$, equation (19) written for $t=0$ gives

$$(2+\beta)[s_2(0)+(1+i^*)A(-1)]/\lambda - w(1+i^*)/\lambda = s_2/\lambda - [B(0)-s_1(0)].$$

Equation (7) written for $t=0$ gives

$$B(0)-s_1(0) = s_2(0)/(1+n)+(1+i^*)B(-1)/(1+n)$$

We eliminate $B(0)-s_1(0)$ between the two last equations. We get

$$s_2 = [2+\beta+\lambda/(1+n)]s_2(0)+(2+\beta)(1+i^*)A(-1)-w(1+i^*)+\lambda(1+i^*)B(-1)/(1+n)$$

We eliminate s_2 between this equation and the first equation of the proof. We get equation (24).

b) If we substitute the expression of $s_2(0)$ given by equation (24) in the last expression of s_2, we get equation (25).

c) If we substitute the expression of $s_2(0)$ given by equation (24) in equation (14), we get equation (26). We use equations (12) and (13) to get equation (27).

d) Equation (19) written for $t \geq 0$ gives

$$B(t)-s_1(t) = -(2+\beta)C_2/\lambda + w(1+i^*)/\lambda + s_2/\lambda$$

If we substitute in this equation, the expressions of s_2 and C_2 given by equations (25) and (26), we get equation (28).

e) Assumption 2 is equivalent to $C_2(t)>0$, which is equivalent to $(1+n)w+(i^*-n)[A(-1)-B(-1)]>0$. This condition is satisfied under Assumption 4.

f) We subtract $B(t)$ from the expression of $A(t)$ given by equation (19)

$$(2+\beta)(1+i^*)[A(t)-B(t)] = w(1+i^*)-(1+\beta)s_2+[(1+i^*)(2+\beta)-\lambda(1+\beta)][s_1(t)-B(t)]$$

We substitute in this equation the expressions of s_2 and $B(t)$ given by equations (25) and (28). We get equation (29).

g) Equation (30) is deduced from equations (22), (25) and (28).□

Proof of Proposition 3

a) The last equation of the proof of Lemma 3 gives equation (38).

b) In the proof of Proposition 1a we established

$$s_2 = [2+\beta+\lambda/(1+n)]s_2(0) + (2+\beta)(1+i^*)A(-1) - w(1+i^*) + \lambda(1+i^*)B(-1)/(1+n)$$

We eliminate s_2 between this expression and equation (38) and get equation (39).

c) If we use equations (14) and (39) we get

$$C_2(0) = \frac{\lambda(1+i^*)}{2+\beta+\lambda/(1+n)}\{w/(i^*-n) + [A(-1)-B(-1)]/(1+n)\}$$

We easily deduce equations (40) and (41) from this expression and equations (12) and (13).

d) Equation (14) and (19) written for $t \geq 0$ give

$$\lambda[B(t) - s_1(t)] = s_2 + w(1+i^*) - (2+\beta)C_2(t+1)$$

If we use the expressions of s_2 given by equation (38), we get equation (42).

e) Assumption 2 is equivalent to $C_2(t) > 0$, which is equivalent to $(1+n)w + (i^*-n)[A(-1)-B(-1)] > 0$. This condition is satisfied under Assumption 4.

f) We deduce equation (43) from equations (22), (38) and (42).

g) We deduce equation (44) from equation (14).□