Chapter 3
The Real Exchange rate and the Terms of Trade

Definitions

- A country's price level is defined as the domestic purchase price, in terms of a *numéraire* (e.g. the dollar), of a well-defined basket of commodities. The reference basket usually represents a bundle of "typical consumer purchases" (the same for all countries). It is practical to choose the reference basket such that its value is one dollar (or 1,000 dollars) in the US. You can also assume that the reference basket is such that its value was one (or 1,000) dollars at a specified date (e.g. January 1st 2005).
- The Penn World Tables publish such price levels. The following graph was derived from these tables

Definitions

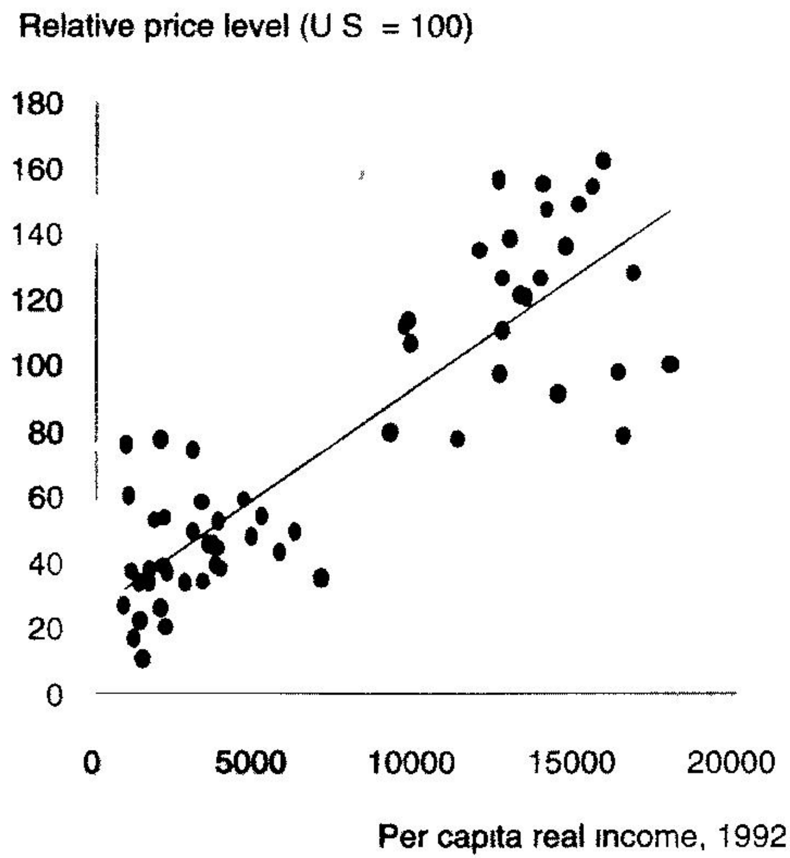

Figure 4.1
Real per capita incomes and price levels, 1992. (*Source:* Penn World Table)

Definitions

- We can see that price levels (approximately) increase with per capita income. This means that life is cheaper in Morocco than in Switzerland.
- The real exchange rate between Brazil and the US is the ratio of the price level in Brazil divided by the price level in the US (many economists prefer to define the RER between Brazil and the US as the price level in the US divided by the price level in Brazil, which introduces a lot of confusion).

Definitions

- Most often, economists do not use the Penn World Tables data. To compute the real exchange rate (RER) of Brazil, they divide the consumption price index in Brazil, by the consumption price index in the US time the price of one dollar in reais. So, they easily get a monthly series of the Brazilian RER.
- As this computation uses indices, its result is a real exchange rate index, which means that it has to be equal to one (or 100) in a reference date (for instance January 2005). Thus, this index determines the changes in the RER and not its level (which can be computed from the Penn World data).

Definitions

- Another problem with this approach is that the baskets of goods used to compute the consumption price index in Brazil and in the US, differ.
- A sophistication is to divide the consumption price index in Brazil by a weighted average of the consumptions price indices in each trade partner of Brazil, time the price of the partner's currency in reais.
- The terms of trade of a country is the ratio of the price index of its exports by the price index of its imports.

Traded and nontraded goods

- In chapters 1 and 2 we assumed that a unique good could be internationally traded freely and without cost (e.g. transport cost). The first good considered in this chapter has this property and is called the traded good. It does not differ between countries and will be taken as *numéraire*.
- The second good introduced in this chapter cannot be internationally traded at any cost. It has to be produced at home. It often is a service, for instance an haircut or a dinner in restaurant. This good is called nontraded good, and there is one of them in each country. Its price (in units of traded good) is p.

Traded and nontraded goods

- The reality is much less clear-cut. Many goods can be internationally traded but at a cost. Goods, which are easily traded nowadays, e.g. wheat, were nontraded in the past, for instance in the 17^{th} century. A good can become nontraded because of the trade policy of some countries.

The model

- We consider a small open economy producing and consuming a traded and a nontraded good. There is no uncertainty (assumption of perfect foresight).
- Each good is produced with capital and labor. Capital is perfectly mobile across the two sectors and with the rest of the world (as in chapter 1). Labor is perfectly mobile across the two sectors, but not with the rest of the world (no migrations).
- Only the traded good can be transformed into capital.
- The traded good is the *numéraire* (for the price of the nontraded good, the wage rate and the world interest rate).

The model. The supply side

- We will omit the time index when there is no ambiguity about the period.
- The production functions of traded and nontraded goods are:

$$Y_T = A_T F(K_T, L_T)$$
$$Y_N = A_N G(K_N, L_N)$$

The two production functions are homogenous of degree 1, increasing in their both arguments and concave.

- Labor mobility insures that workers earn the same wage w in either sector.
- The total domestic labor supply is fixed at

$$L = L_T + L_N$$

- The world interest rate is r and the price of the non-traded good is p.

The model. The supply side

- The value of the firms of each sector at the beginning of period 0 are:

$$V_{T,0} = \sum_{t=0}^{\infty} \frac{A_{T,t}F\left(K_{T,t}, L_{T,t}\right) - w_t L_{T,t} - \left(K_{T,t+1} - K_{T,t}\right)}{(1+r)^{t+1}}$$

$$V_{N,0} = \sum_{t=0}^{\infty} \frac{p_t A_{N,t}G\left(K_{N,t}, L_{N,t}\right) - w_t L_{N,t} - \left(K_{N,t+1} - K_{N,t}\right)}{(1+r)^{t+1}}$$

  - Assume that firms are entirely financed by their owners. The numerators of the fractions of the right-hand sides of the two expressions are the cash flow obtained by these owners at the end of period t. The sums of the present values of all the future cash flows are the values of the firms in each sector.

The model. The supply side

- The owners of the firms set their demand of labor to maximize their values. We obtain the foc:

$A_{T,t}F_L{}'(K_{T,t}, L_{T,t}) = w_t = p_t A_{N,t} G_L{}'(K_{N,t}, L_{N,t})$ for $t \geq 0$

- The owners of the firms set their demand of capital to maximize their values. We obtain the foc :

$\frac{\partial V_{T,0}}{\partial K_{T,t}} = 0$, for $t \geq 1$ because $K_{T,0}$ is inherited from the past (we assume that capital can only be changed at the end of a period).

The model. The supply side

$$\frac{A_{T,t}F_K{}'\left(K_{T,t},L_{T,t}\right)+1}{(1+r)^t}-\frac{1}{(1+r)^{t-1}}=0$$

or

$$A_{T,t}F_K{}'\left(K_{T,t},L_{T,t}\right)=r=p_tA_{N,t}G_K{}'\left(K_{N,t},L_{N,t}\right)$$

for $t\geq 1$

The model. The supply side

- Let us define the capital-labor ratios in the tradable and nontradable sectors as $k_T \equiv K_T/L_T$ and $k_N \equiv K_N/L_N$.
  - The outputs per worker are

$$y_T = A_T f(k_T) \equiv A_T F(k_T, 1) \equiv A_T F(K_T, L_T)/L_T$$

$$y_N = A_N g(k_N) \equiv A_N G(k_N, 1) \equiv A_N G(K_N, L_N)/L_N$$

  - Differentiate the first equation

$$A_T f(K_T/L_T)L_T \equiv A_T F(K_T, L_T)$$

$$A_T f'(k_T)dK_T + [A_T f(k_T) - A_T f'(k_T)k_T]dL_T = rdK_T + wdL_T$$

The model. The supply side

- We can rewrite the foc in the traded good sector:

$$A_{T,t}f\left(k_{T,t}\right) - rk_{T,t} = w_t, \text{ for } t \geq 0 \qquad (1)$$

$$A_{T,t}f'\left(k_{T,t}\right) = r, \text{ for } t \geq 1 \qquad (2)$$

- In the nontraded good sector we have:

$$p_t A_{N,t}g\left(k_{N,t}\right) - rk_{N,t} = w_t, \text{ for } t \geq 0 \qquad (3)$$

$$p_t A_{N,t}g'\left(k_{N,t}\right) = r, \text{ for } t \geq 1 \qquad (4)$$

The model. The supply side

- Assume that we are in period t with $t \geq 1$. We use the four foc for this period to compute its equilibrium. We simplify the notations by removing the time subscripts
  - Eq. 2 gives: $k_T(r/A_T) = f'^{-1}(r/A_T)$. As $f"(k_T) < 0$, the capital labor ration in the tradable sector is a decreasing function of the world interest rate.
  - We substitute this expression of k_T in Eq. 1 and get $w_t(r, A_T) = A_{T,t} f\left(k_T(r/A_T)\right) - r k_T(r/A_T)$.

The model. The supply side

- This last equation is the *factor-price frontier* relationship in the tradable sector. We differentiate this equation and get

$$\partial w_T/\partial r = A_T f'(k_T)\frac{\partial k_T}{\partial r} - k_T - r\frac{\partial k_T}{\partial r} = - k_T < 0$$

The wage rate is a decreasing function of the world interest rate.

- We substitute $w(r, A_T)$ for w in Eq. 3. This and Eq. 4 jointly determine k_N and p. The figure is a graphical display of the solution, which can be denoted $k_N(r, A_T, A_N)$ and $p(r, A_T, A_N)$.

The model. The supply side

- The schedule labeled **MPK** graphs Eqs. 4, $pA_N g'(k_N) = r.$ Its slope is upward because a rise in the price of nontradable raises the marginal value product of capital and, given *r*, the optimal capital intensity of production
- The schedule labeled **MPL** can be written as

$$pA_N[g(k_N) - g'(k_N)k_N] = w(r, A_T)$$

MPL has a negative slope because, given *r* and, hence, *w*, a higher k_N raises the marginal physical product of labor, equal to $A_N[g(k_N) - g'(k_N)k_N]$. The price *p* must therefore fall to keep labor's marginal value equal to its level $w(r, A_T)$ in the traded sector.

The model. The supply side

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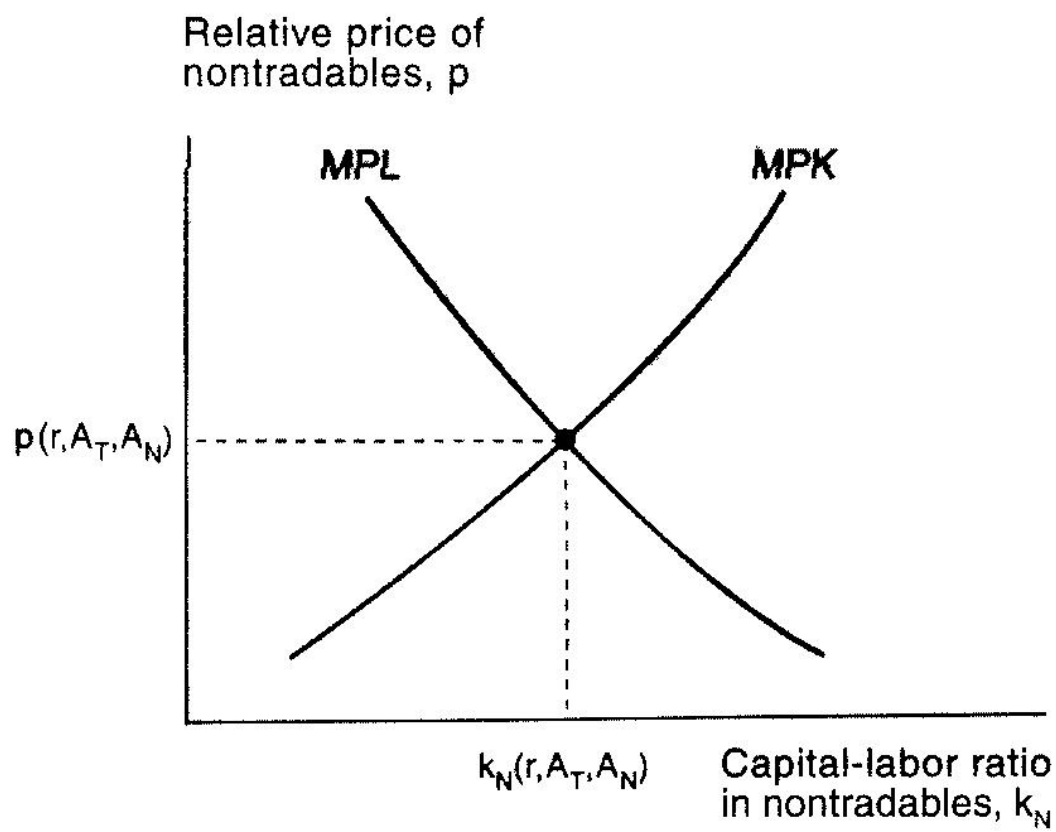

Figure 4.2
Determination of the price of nontradables

The model. The supply side

- An important result is that the price of the nontraded good, the wage rate and the capital intensities in both sectors are independent of the consumer's demand patterns.
- However, we will see that the allocation of labor between the two sectors, and so their relative outputs depend on the consumer's preference.

The model. The value of firms

- Rewrite the four previous equations in any period t, with $t \geq 1$. We don't need to specify this period:

$$A_T f(k_T) - r k_T - w = 0 \qquad (1)$$

$$A_T f'(k_T) = r \qquad (2)$$

$$p A_N g(k_N) - r k_N - w = 0 \qquad (3)$$

$$p A_N g'(k_N) = r \qquad (4)$$

The profit of the firms in the tradable sector are

$$A_T F(K_T, L_T) - w L_T - r K_T = [A_T f(k_T) - r k_T - w] L_T = 0$$

Hence, Eqs. 1 and 3 are non profit conditions for the two sectors.

The model. The value of firms

- We can rewrite

$$V_{T,0} = A_{T,0} F\left(K_{T,0}, L_{T,0}\right) - w_0 L_{T,0} - \left(K_{T,1} - K_{T,0}\right) + \frac{V_{T,1}}{1+r}$$

with

$$V_{T,1} = \sum_{t=1}^{\infty} \frac{A_{T,t} F\left(K_{T,t}, L_{T,t}\right) - w_t L_{T,t} - \left(K_{T,t+1} - K_{T,t}\right)}{(1+r)^t}$$

- We deduce from the nonprofit condition in the T sector that the values of its firms at the beginning of period 1, is

$$V_{T,1} = K_{T,1}$$

Then, the value of the firms is equal to the cost of their capital

The model. The value of firms

- We also have

$$V_{T,0} = \left[F\left(K_{T,0}, L_{T,0}\right) - w_0 L_{T,0} + K_{T,0}\right]/(1+r)$$

which in general differs from $K_{T,0}$

- In the model, it takes one period to install capital. Then, under the assumption of perfect foresight, capital is always at its optimum level from the beginning of period 1. However, in period 0 capital is inherited from the past and is not at its optimal level.

The model. The value of firms

- We have a similar difference between the value of firms and the cost of their capital in every periods where consumers revise their expectations.
- In models where the installation of capital takes more than one period, the difference between the value of firms and the cost of their capital lasts more than one period.
- The ratio between the value of firms and the cost of their capital is called the *Tobin q*. It should be one in the long run, but may differ from one at shorter horizons.

Price effects of anticipated productivity and interest rate shifts

- Differentiate Eq.1

$$f(k_T)dA_T + A_T f'(k_T)dk_T - rdk_T - k_T dr - dw$$
$$= f(k_T)dA_T - k_T dr - dw = 0$$

Denote the relative change in variable x by $\hat{x} = dx/x$.

Let $\mu_{LT} = \frac{wL_T}{Y_T} = \frac{w}{A_T f(k_T)}$ be the labor's share of the income generated in the traded good sector. Then, the capital share is:

$\frac{rK_T}{Y_T} = \frac{rk_T}{A_T f(k_T)} = 1 - \mu_{LT}$. We have

$$A_T f(k_T)\widehat{A}_T - rk_T\hat{r} - w\widehat{w} = 0$$

or

$$\widehat{A}_T - (1 - \mu_{LT})\hat{r} = \mu_{LT}\widehat{w}$$

Price effects of anticipated productivity and interest rate shifts

- Differentiate Eq. 3:

$$\hat{p} + \widehat{A}_N - (1 - \mu_{LN})\hat{r} = \mu_{LN}\widehat{w}$$

- Substitute the value $\widehat{w}$ given by the first equation in the second equation. We get

$$\hat{p} = \frac{\mu_{LN}}{\mu_{LT}}\widehat{A}_T - \widehat{A}_N + \left(1 - \frac{\mu_{LN}}{\mu_{LT}}\right)\hat{r}$$

or

$$\hat{p} = \left(\frac{\mu_{LN}}{\mu_{LT}} - 1\right)\widehat{A}_T + \left(\widehat{A}_T - \widehat{A}_N\right) \quad + \left(1 - \frac{\mu_{LN}}{\mu_{LT}}\right)\hat{r}$$

Price effects of anticipated productivity and interest rate shifts

- We assume that the labor's share is higher in the nontraded than in the traded good sector $\mu_{LN} \equiv \frac{w}{pA_N g(k_N)} > \mu_{LT} \equiv \frac{w}{A_T f(k_T)}$. With the first and third equation this condition is equivalent to $rk_N + w < rk_T + w$ or $k_N < k_T$.
- That is the non-traded good sector is more labor intensive (and less capital intensive) than the traded good sector. This condition holds in practice. The nontradable sector substantially overlaps with services, which are inherently less susceptible to standardization and mechanization than are manufactures or agriculture.

Price effects of anticipated productivity and interest rate shifts

- Faster productivity growth in tradables than in nontradables will push the price of nontradables over time. The "Baumol-Bowen effect" of a rising relative price of services comes through very clearly in the data as time-series evidence for industrial countries show (next figure).
- Across industrial countries, there is also a positive cross-sectional relation between long-run tradables-nontradables productivity-growth differentials and long-run rates of increase in p (see figure).
- A rise in the world interest rate lowers the relative price of the non-traded good. We will investigate the dramatic consequences of this result in the chapter on the sudden stop of capital inflows.

Price effects of anticipated productivity and interest rate shifts

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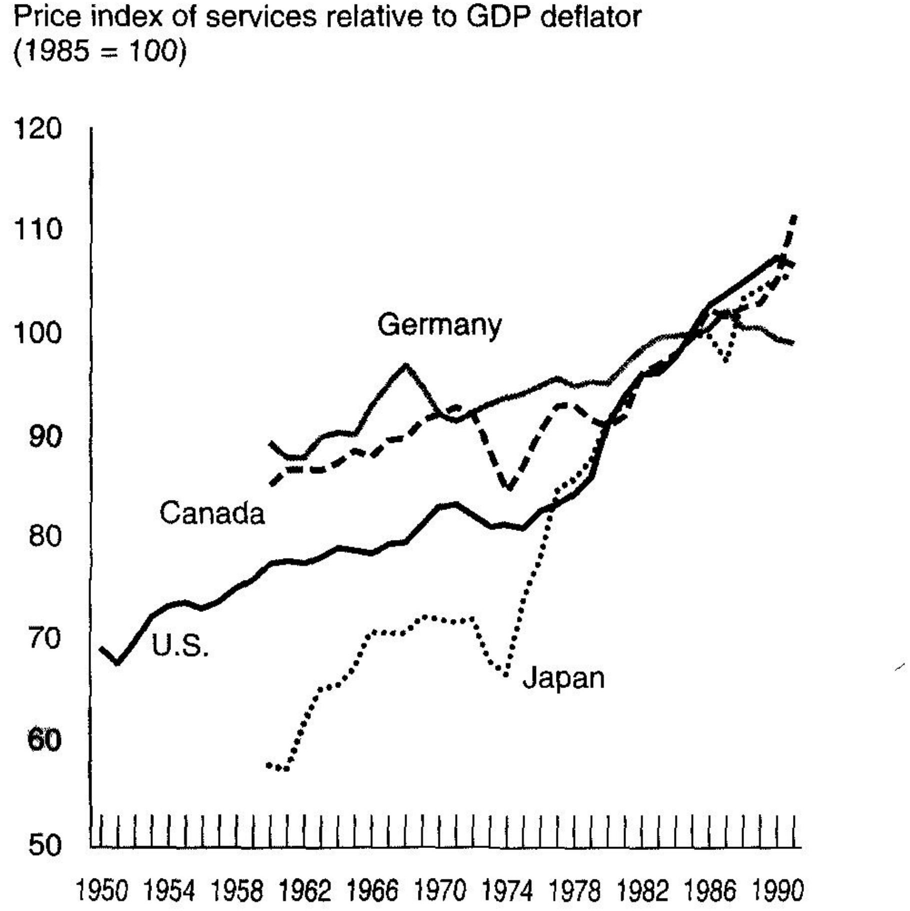

Figure 4.3
The relative price of services

Price effects of anticipated productivity and interest rate shifts

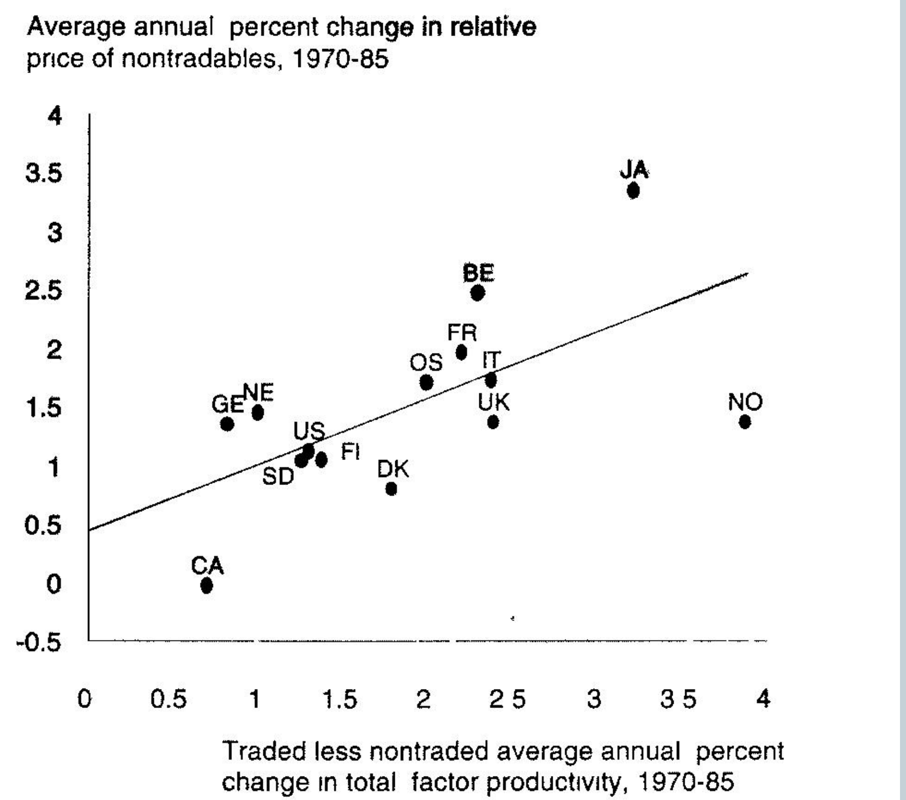

Figure 4.4
Differential productivity growth and the price of nontradables. (*Source:* Wolf, 1994)

The Harrod-Balassa-Samuelson effect

- The Harrod-Balassa-Samuelson effect is a tendency for countries with higher productivity in tradables compared with nontradables to have higher price levels.
- We assume that traded goods are a composite with a uniform price in each of two countries, Home and Foreign. Nontraded goods have distinct Home and foreign prices in terms of tradables, denoted p and $p*$. We assume that the price level in each country is a geometric average, with weights γ and $1-\gamma$, of the prices of tradables and nontradables.

$P = 1^{\gamma}p^{1-\gamma} = p^{1-\gamma}$ and $P *= 1^{\gamma}p *^{1-\gamma} = p *^{1-\gamma}$

The Harrod-Balassa-Samuelson effect

- The Home-to-Foreign price level ratio is

$$\frac{P}{P*} = \left(\frac{p}{p*}\right)^{1-\gamma}$$

Home's real exchange rate against Foreign depends only on the internal relative prices of nontraded goods.

- Differentiate this equation:

$$\widehat{P} - \widehat{P}* = (1-\gamma)(\hat{p} - \hat{p}*)$$

Assume that both countries' sectoral outputs be proportional to the same functions $F(K_T, L_T)$ and $G(K_N, L_N)$, but with possibly different factor productivities. Then, the labor's share in each sector is the same in both countries.

The Harrod-Balassa-Samuelson effect

- Finally, we deduce from slide 26

$$\widehat{P} - \widehat{P}$$
$$= (1-\gamma)\left[\left(\frac{\mu_{LN}}{\mu_{LT}} - 1\right)\left(\widehat{A}_T - \widehat{A}_T *\right) + \left(\widehat{A}_T - \widehat{A}_T *\right)\right.$$
$$\left. - \left(\widehat{A}_N - \widehat{A}_N *\right)\right]$$

- It follows that Home will experience real appreciation if its productivity-growth advantage in tradable exceeds its productivity-growth advantage in nontradables.

The Harrod-Balassa-Samuelson effect

- As productivity growth is higher in tradables than in nontradables, rich countries have become rich mainly through high productivity in tradables. We obtain the famous prediction of the Harrod-Balassa-Samuelson proposition, that price levels tend to rise with country per capita income (first figure of this chapter).

Consumption and production in the long run

- All the results we have reached so far are independent of demand. We have seen (slide 21) that capital intensity in the two sectors k_T and k_N, the wage rate w and the price of the nontraded good p are functions of the world interest rate r and the productivities of both sectors A_T and A_N .
- We will assume in the following slides that in the long run, these variables have set to their long run values. We will look for the stationary equilibrium (the steady state) of the economy, which can be interpreted as its long run equilibrium. We saw that a necessary condition for the existence of a steady state is that $(1 + r)\beta = 1$.

Consumption and production in the long run

- We derive from the non profit conditions (Eqs. 1 and 3) and the full employment assumption:

$$Y_T = A_T f(k_T) L_T = (rk_T + w) L_T$$
$$pY_N = pA_N g(k_N) L_N = (rk_N + w) L_N$$
$$L_T + L_N = L$$

- We deduce from these three equations the expression of the output of traded good in function of the output of nontraded good

$$Y_T = (rk_T + w)L - \frac{rk_T + w}{rk_N + w} pY_N$$

Consumption and production in the long run

- Consumer income is equal to the country GDP plus its income on foreign lending

$$R = Y_T + pY_N + rB$$

$$\text{or } R = (rk_T + w)L - \frac{r(k_T - k_N)}{rk_N + w} pY_N + rB$$

- The consumer lifetime utility is the same as in chapter 2

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

Consumption and production in the long run

- C is a consumption index, which aggregates the consumption of T and N goods with the Cobb Douglas function: $C = (C_T/a)^a[C_N/(1-a)]^{1-a}$, with $0 < a < 1$.
- The maximization of the consumption index under the budget constraint $C_T + pC_N = R$, determines the demand of nontraded good: $pC_N = (1-a)R$ and of traded good $C_T = aR$. Then, $p^{1-a}C = R$.
- p^{1-a} can be interpreted as the consumption price index. It is a geometric mean between the price of tradable and nontradable. As the rest of the world consumption price is assumed fixed, p^{1-a} can also be interpreted as the RER.

Consumption and production in the long run

- The equilibrium on the nontraded good market is

$$C_N = Y_N$$

$$\text{or } (1-a)R = pY_N,$$

or

$$(1-a)\left[(rk_T + w)L - \frac{r(k_T - k_N)}{rk_N + w} pY_N + rB\right] = pY_N$$

- Finally, the production of nontraded good is given by:

$$\left[\frac{a}{1-a} + \frac{rk_T + w}{rk_N + w}\right] pY_N = (rk_T + w)L + rB$$

Consumption and production in the long run

- The consumption of traded good is

$$C_T = \frac{a}{1-a} pC_N = \frac{a}{1-a} pY_N$$

- The trade balance is

$$T = Y_T - C_T = (rk_T + w)L - \left(\frac{rk_T + w}{rk_N + w} + \frac{a}{1-a}\right) pY_N = -rB$$

- The current account is balanced in each period:

$$CA = T + rB$$

Hence, the stock of foreign assets remains the same overtime.

- Finally, there exists an infinity of stationary equilibriums, parameterized by the stock of foreign assets B.

The terms of trade and the real exchange rate

- So far we have simplified by assuming that there is a single traded composite good. In reality, the goods a country exports tend to differ from those it imports, and the relative price of imports and exports – the terms of trade – changes over time. Terms-of-trade changes affect the whole equilibrium of the economy, and so the RER and the current account.
- Exports in most developing countries are often concentrated on some key commodities or agricultural products. Commodity prices are extremely volatile.

The terms of trade and the real exchange rate

- I will assume that the country exports a quantity X of a commodity, which is produced without requiring any input (labor or capital). The price of this commodity (in units of tradable, which is the *numéraire*) is p^*. X and p^* are exogenous and can change over time. The country imports tradable, and so p^* is also the terms of trade.
- The new flow of income generated by commodity exports, p^*X, can be introduced in the model just by adding it to the consumer's income of each period.
- These exports have no effect on the capital intensity in tradable and nontradable, the wage rate w and the price of the nontraded good p that is on the real exchange rate.

The terms of trade and the real exchange rate

- This last result is unrealistic: we would have expected an appreciation of the RER.
- The higher consumer's income induces a higher demand of traded and nontraded goods. As the latter good is produced locally, its production and its price should increase. As labor moves from tradable to nontradable, the output of traded good decreases, and so imports increase. The contraction of the tradable sector (Dutch disease) is realistic. With the previous model I get the latter result (Dutch disease) bur not the former result (no appreciation of the RER)!

The terms of trade and the real exchange rate

- CV (chapter 4) considers a model with the same three sectors, but where the endowment path of each good is exogenously given. Hence, by construction, a rise in commodity exports has no effect on the output of tradable (no Dutch disease). However, higher consumer's income and demand induce an appreciation of the RER.
- This suggests to develop an intermediary model, where the stock of capital in the tradable and nontradable sectors would be fixed and exogenous (capital is immobile between sectors), but with labor free to move between these sectors.

The terms of trade and the real exchange rate

- The economic interpretation of these three models:
  - The model where labor and capital can freely move between sectors, and move internationally for the latter, developed until now in this chapter, gives a good representation of the economic long run. Hence, it was useful to interpret the Harrod-Balassa-Samuelson effect.
  - The model with fixed endowments represents the short run, when labor and capital have not still moved where their return is the highest
  - The model with endogenous labor represents the medium run: labor can freely move, but not capital.

The terms of trade and the real exchange rate: the demand side

- The demand side of the model is very similar to what we developed earlier in the chapter. To simplify the exposition, we will assume that the initial net stock of foreign assets is zero: $B_0 = 0$. We will also assume until when we consider the supply side that the outputs of tradable, nontradable and commodity remain constant over time: Y_T , Y_N and X. The commodity price $p_t *$ can change over time.
- We will also assume that consumers have logarithmic preferences: $u(.) = log(.)$ and that: $(1 + r)\beta = 1$

The terms of trade and the real exchange rate: the demand side

- The consumer's income and spending are:

$$R_t = Y_T + p_t Y_N + p_t * X + rB_t$$

$$S_t = R_t - (B_{t+1} - B_t) = C_{T,t} + p_t C_{N,t} = {p_t}^{1-a} C_t$$

  - ${p_t}^{1-a}$ is the consumption price index and the RER. C_t is the consumption index. The consumer welfare at the beginning of period zero is:

$$U_0 = \sum_{t=0}^{\infty} \beta^t log(C_t) = \sum_{t=0}^{\infty} \beta^t log\{{p_t}^{-1+a}[Y_T + p_t Y_N + p_t * X + (1+r)B_t - B_{t+1}]\}$$

The terms of trade and the real exchange rate: the demand side

- The consumer maximizes U_0 in $B_{t+1}, t \geq 0$. The foc are:
$$\frac{\partial U_0}{\partial B_{t+1}} = 0 \text{ or } -\frac{\beta^t p_t{}^{-1+a}}{C_t} + \beta^{t+1}\frac{(1+r)p_{t+1}{}^{-1+a}}{C_{t+1}} = 0.$$
Finally: $p_{t+1}{}^{1-a}C_{t+1} = p_t{}^{1-a}C_t$
or $S_t = S$, constant for $t \geq 0$.
- Remember that the consumption of traded and nontraded goods are: $C_{T,t} = aS$ and $p_t C_{N,t} = (1-a)S$. Moreover, the consumption of nontraded good is equal to its output: $C_{N,t} = Y_N$. Hence, the price on nontraded good is constant over time: $p_t = p = \frac{(1-a)S}{Y_N}$.

The terms of trade and the real exchange rate: the demand side

- The consumption of traded good is perfectly smoothed that is constant over time.
- The identity of the current account balance is:

$$CA_t = Y_T + p_t * X + rB_t - C_T = B_{t+1} - B_t, t \geq 0$$

- As the exports of commodity, $p_t * X$, is positive, if $B_{t+1} - (1 + r)B_t$ is not too large, the country will be a net importer of tradable. Then, $p_t *$ is the terms of trade.

The terms of trade and the real exchange rate: the demand side

- If we use the transversality condition : $\lim_{T\to\infty} B_{T+1}/(1+r)^{T+1}$, and the assumption that the initial stock of foreign assets B_0 is zero we have

$$\sum_{t=0}^{\infty} \frac{Y_T + p_t * X}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{C_T}{(1+r)^t}$$

or

$$aS = C_T = Y_T + r\sum_{t=0}^{\infty} \frac{p_t * X}{(1+r)^{t+1}}$$

We deduce from this expression the price of the nontraded good, the consumption of traded good, and the dynamic path of foreign assets.

The terms of trade and the real exchange rate: the demand side

- We compute a benchmark equilibrium where the commodity price is constant, $p*$. The expression of the consumption of traded good, the price of the nontraded good, the current account surplus and the stock of foreign assets are

$$C_T = Y_T + p*X, \quad p = \frac{1-a}{a}\frac{Y_T + p*X}{Y_N}$$

$$CA_t = B_t = 0, \; t \geq 0.$$

- We assume that the commodity price increases by $dp*> 0$ for $0 \leq t \leq T$, and then come back at its initial value for $t \geq T + 1$.

The terms of trade and the real exchange rate: the demand side

- The consequence of this *transitory* increase in the commodity price is a *permanent* increase in consumer spending, the consumption of traded good and the price of nontraded good:

$$adS = dC_T$$

$$= r\sum_{t=0}^{T} \frac{Xdp*}{(1+r)^{t+1}} = Xdp*\left(1 - \frac{1}{(1+r)^{T+1}}\right)$$

$$dp = \frac{(1-a)Xdp*}{aY_N}\left(1 - \frac{1}{(1+r)^{T+1}}\right)$$

The terms of trade and the real exchange rate: the demand side

- The current account balance turns a surplus for $0 \leq t \leq T$:

$$dCA_t = dB_{t+1} - dB_t = Xdp * + rdB_t - dC_T$$

$$= rdB_t + \frac{Xdp *}{(1+r)^{T+1}}$$

Hence the stock of foreign assets increases over time:

$$dB_{t+1} = \frac{Xdp *}{(1+r)^{T+1}} \frac{(1+r)^{t+1} - 1}{r}$$

The terms of trade and the real exchange rate: the demand side

- For $t \geq T+1$, the current account surplus is

$$dCA_t = dB_{t+1} - dB_t = rdB_t - dC_T$$

$$= rdB_t - Xdp*\left(1 - \frac{1}{(1+r)^{T+1}}\right)$$

with

$$dB_{T+1} = \frac{Xdp*}{r}\left[1 - \frac{1}{(1+r)^{T+1}}\right]$$

  - Hence

$$dCA_t = dB_{t+1} - dB_t = r(dB_t - dB_{T+1}) = 0$$

The stock of foreign assets remains constant over time, and the current account balance is in equilibrium.

The terms of trade and the real exchange rate: the demand side

- Finally a *transitory* increase in the commodity price that is in the term of trade:
  - Permanently raises the consumption of traded good and the price of the nontraded good. That is the RER permanently appreciates .
  - During the transitory rise in the commodity price, the country runs a surplus of its current account balance and accumulates foreign assets.
  - When the commodity price is back to its initial level, the current account balance comes back to equilibrium, but the capital income on foreign assets allows the country to permanently run a trade balance deficit.

The terms of trade and the real exchange rate: the supply side

- The production functions of each sector are:

$$Y_T = A_T {L_T}^{\alpha}, \text{ with } 0 < \alpha < 1$$

$$Y_N = A_N L_N$$

- The assumption of constant labor productivity in the *N* sector is very strong and probably unrealistic. However, it will simplify the math. The labor shares in the income produced by the two sectors respectively are equal to α and 1. This share is higher in the *N* sector as was assumed before.
- We also have full employment of labor:

$$L = L_T + L_N$$

The terms of trade and the real exchange rate: the supply side

- Firms set their demand of labor to satisfy the marginal conditions:

$$\frac{\partial Y_T}{\partial L_T} = \alpha A_T {L_T}^{\alpha-1} = w, \;\; p\frac{\partial Y_N}{\partial L_N} = pA_N = w$$

  - So $L_T = \left(\frac{\alpha A_T}{pA_N}\right)^{1/(1-\alpha)}$ and $L_N = L - L_T$
  - The permanent rise in the price of the nontraded good, induced by the transitory rise in the commodity price, induces a permanent and constant increase in the wage rate:

$$dw/w = dp/p$$

The terms of trade and the real exchange rate: the supply side

- Employment in the traded good sector and its output permanently decrease by fixed amounts:

$$\frac{dL_T}{L_T} = -\frac{1}{1-\alpha}\frac{dp}{p} < 0, \frac{dY_T}{Y_T} = -\frac{\alpha}{1-\alpha}\frac{dp}{p} < 0$$

- Employment in the nontraded good sector and its output permanently increase by fixed amounts

$$\frac{dY_N}{Y_N} = \frac{dL_N}{L_N} = \frac{-dL_T}{L_N} = \frac{L_T}{L_N}\frac{\alpha}{1-\alpha}\frac{dp}{p} > 0$$

- The real cost of labor in the traded good sector: $w/1$ increases. In the nontraded good sector this cost w/p remains unchanged (it would decrease if marginal labor productivity was decreasing in this sector).

The terms of trade and the real exchange rate: the supply side

- The movements in outputs lead to corrections of the previous results.
- First, the production of traded good permanently decreases. This deindustrialization, often called *Dutch disease*, reduces the size of the trade balance surplus during the $T+1$ periods of higher commodity price, and so the accumulated foreign assets at the end of this period.
- The production of nontraded good permanently increases. Thus, the rise in the nontraded good price and the appreciation of the real exchange rate are slowed down.

The terms of trade and the real exchange rate: the supply side

- Proving that the trade balance still runs a transitory surplus and that the RER still appreciates that is that the supply side of the model does not change the qualitative results derived from the demand side, is a little cumbersome but easy.
- All the other results are qualitatively unchanged.
- In conclusion, an increase in commodity price raises consumer income and its demand. The imports of traded good and the price of the nontraded good increase. This leads to a displacement of labor from the tradable to the nontradable sector. Hence the output of nontradable increases and the output of tradable decreases (*Dutch disease*).

The twin deficits hypothesis

- CV (chapter 4) introduces government's consumption in the same model. If government spending is biased toward nontradable goods (relative to the private sector's spending) then a transitory increase in government spending (which creates a budget government deficit) crowds out consumer's demand from nontraded good and increases its price. The RER appreciates during the period of the shock.
- This rise in price induces an increase in the production of nontraded good with a transfer of labor from tradable to nontradable sector.

The twin deficits hypothesis

- The decrease in the output of traded good induces temporary trade and current account deficits: fiscal deficits cause trade deficits, the so-called *twin deficits* hypothesis.
- CV quotes a series of econometric studies, which conclude that in Latin America and the OECD countries there is a positive impact of government spending on real exchange rate.
- In the studies conducted for Asia, no significant relationship has been found. An interpretation is that Asian governments have a stronger preference for spending on tradable goods. The share of public investment in GDP – a component intensive in tradable goods – is systematically higher in Asia than in Latin America.