CHAPTER 4. AN EXERCISE IN COMPARATIVE STATIC

1. Theory

The general structure of a *static model* can be written

(1) $F(y, x, a) = 0$

- $F(\ )$ is a vector of n equations
- y is a vector of n endogenous variables
- x is a vector of m exogenous variables
- a is a vector of p parameters

Simulating the model is solving a system of n equations with n unknown variables, for given values of the exogenous variables and of the parameters.

Before simulating the model, we must set the values of the parameters and of the exogenous variables. The model should be able to reproduce the average situation of the economy observed on a period of several years. This average situation of the economy is defined by the values of the endogenous and exogenous variables, which are observable (e.g. GDP is observable, but not the total productivity of factors). So we want the equations of the model to be satisfied when some endogenous and exogenous variables haves been set at their observed values

(2) $F(y, x, a) = 0$

Moreover, the values of some parameters are known (for instance they were computed by an econometric estimation).

There are n equations in system (2), and $n+m+p$ exogenous variables, endogenous variables and parameters. So, we can set the values of $m+p$ of these variables and parameters, and use system (2) to compute the values of the other variables and parameters.

Thus, to calibrate a model we have to solve a system of *n* equations with *n* variables, which is mathematically the same thing as simulating a static model. At the end of the calibration stage we have defined a reference state of the economy. We will do our static comparative exercises around this reference state.

2. An example of a static model

I will use the model developed in: "Environmental Regulations for a Small Open Economy with Tourism", by Chi-Chur Chao, Bharat R. Hazari, Jean-Pierre Laffargue and Eden S. H. Yu, in *Globaliation and Emerging Issues in Trade Theory and Policy*, Binh Tran-Nam, Ngo Van Long and Makoto Tawada, eds., Emerald, 2008, pp.269-284.

2.1. Equations (16)

The model represents a small country, which produces three goods: an agricultural good X, a manufacturing good Y and a non traded good N. Good X is exported and good Y is imported. Each sectors uses labour, which is a perfectly mobile factor, and a specific immobile factor. The total supply of labour L is fixed. Equations (1) to (3) are the production functions of the three sectors. Equation (4) is the equilibrium of the labour market, which establishes the equality between the total quantity of labour and the sum of the demands of labour by the three sectors.

(1) $X = AL_X^{\alpha}$

(2) $Y = BL_Y^{\beta}$

(3) $N = CL_N{}^{\gamma}$

(4) $L = L_X + L_Y + L_N$

Equation (5) links the domestic price of imports, p, to their foreign price, $p*$, which is exogenous, and to the tariff rate, t.

(5) $p = p*+t$

Equations (6), (7) and (8) are the marginal conditions in sectors X, Y and N, which establishes the equality of the marginal productivity of labour to the real wage rate.

(6) $\alpha X = wL_X$

(7) $\beta(p-s)Y = wL_Y$

(8) $\gamma qZ = wL_N$

Equation (9) establishes that the amount of pollution in the economy is equal to the output of manufacturing good

(9) $Z = Y$

The utility of domestic residents, without taking into account the quality of the environment is given by equation (10). There is a constant elasticity of substitution, equal to $\sigma + 1$, between the non traded good and the manufacturing good. There is an elasticity of substitution of 1 between the aggregate of these two goods and the agricultural good. This utility has the dimension of a consumption index. The utility of households, including the quality of the environment, is given by equation (11). The domestic prices of the three consumption goods respectively are 1, p and q. Equations (12) and (13) are the marginal conditions of the maximisation program of households.

(10) $$V(C_X, C_Y, C_N) = C_X^a \left[b^{1/(1+\sigma)} C_Y^{\sigma/(1+\sigma)} + \bar{b}^{1/(1+\sigma)} C_N^{\sigma/(1+\sigma)} \right]^{\bar{a}(1+\sigma)/\sigma} / \left[b^{1/(1+\sigma)} + \bar{b}^{1/(1+\sigma)} \right]^{\bar{a}(1+\sigma)/\sigma}$$

(11) $$U(C_X, C_Y, C_N) = V(C_X, C_Y, C_N) \left(\frac{\bar{Z}}{\bar{Z} + Z} \right)^d$$

(12) $$C_X = \frac{aqC_N}{\bar{a}} \left[1 + \frac{b}{\bar{b}} \left(\frac{q}{p} \right)^{\sigma} \right]$$

(13) $$C_Y = \frac{b}{\bar{b}} \left(\frac{q}{p} \right)^{1+\sigma} C_N$$

Domestic residents consume the three goods. Tourists only consume non traded good. Equation (14) is the consumption of non traded good by tourists. The price elasticity of this demand function is equal to η.

(14) $D_N = Tq^{-\eta}$

Equation (15) is the equilibrium of the non-traded good market. On the left-hand side of equation (16) we successively find, the exports of agricultural good, minus the imports of manufactured good, plus tourism receipts. The total is the surplus of the balance of trade in goods and invisible, which is zero in the model. The balance of payments identity uses international prices and the agricultural good X as a numeraire.

(15) $N = C_N + D_N$

(16) $(X - C_X) - p^*(C_Y - Y) + qD_N = 0$

The model includes 16 equations and of course, as we will see, 16 endogenous variables.

We also must check the constraints that good X is exported and good Y is imported: $EX = X - C_X \geq 0$ and $M = C_Y - Y \geq 0$.

Finally, Pigou established that in an economy without other distortion the optimal pollution tax rate is equal to the direct marginal rate of pollution that is $s^P = d(C_X + pC_Y + qC_N)/(\overline{Z} + Z)$. We will see that because of the distortion induced by the imports tariff, then the optimal pollution tax rate differs from its Pigovian value.

2.2. Endogenous variables (16)

X : production of agricultural good

Y : production of manufacturing good

Z : amount of pollution

N : production of non-traded good

L_X : employment in the agricultural good sector

L_Y : employment in the manufacturing good sector

L_N : employment in the non-traded good sector

p : domestic price of good Y (good X is the numeraire)

q : price of good N (good X is the numeraire)

w : wage rate (good X is the numeraire)

C_X : domestic households' consumption in agricultural good

C_Y :domestic households' consumption in manufacturing good

C_Z : domestic households' consumption in non-traded good

D_N : tourists' consumption (all in non traded good)

U : current utility of households, effects of the environmental externality included

V : current utility of households, effects of the environmental externality excluded

2.3. Exogenous variables

L : labour supply

s : pollution tax rate; the pollution tax is paid by the producers of good Y

$p*$: foreign price of good Y (good X is the numeraire)

t : imports' tariff rate

T: shift parameter in tourists' demand of non-traded good

2.4. Parameters

$A, B, C > 0$: global productivity of the three sectors

$0 < \alpha < 1$: share of labour income in the production of the agricultural good sector

$0 < \beta < 1$: share of labour income in the production of the manufacturing good sector

$0 < \gamma < 1$: share of labour income in the production of the non traded good sector

$a, b, \overline{a}, \overline{b} > 0$: parameters of the households' utility function. We also have $a + \overline{a} = b + \overline{b} = 1$

$\eta > 0$: price elasticity of the demand of non-traded good by tourists.

$\sigma > -1$: $1 + \sigma$ is the elasticity of substitution between the consumption of non-traded good and of manufacturing good.

$\overline{Z} > 0$, $d > 0$: parameters of the externality in the utility of consumers

3. Calibration

We start by setting the values of the main endogenous variables of the reference equilibrium of the model

X=2; Y=2; N=6; LX=2; CY=3.5; CN=5; ps=1; q=1; t=0; s=0;

and the values of the elasticity parameters

alfa=0.6; beta=0.6; gama=0.6; $\eta = 0.5$; $\sigma = 3$ (elasticity of substitution of 4 between the manufactured good and the non-traded good in the utility function);

Then, we compute the values of the other endogenous or exogenous variables

p=1; DN=1; w=0.6; LY=.2; LN=6; Z=2; L=10; CX=1.5;

the values of the scale parameters

T=1; A=1.3195; B=1.3195; C=2.0477;

and the values of the last parameters

b=0.4118; a=0.15;

We compute the values of the exports and the imports and check that they are positive. We also compute the value of households' utility excluding the effect of the quality of the environment

EX=0.5; M=1.5; V=3.6476;

Finally, we assume that, in the reference steady state, pollution reduces households' utility by 2%. If pollution were doubled, households' utility would be reduced by 5%. Then, the parameters $\overline{Z}$ and d are the solutions of the system of two equations

$$\frac{\overline{Z}}{\overline{Z}+2} = 0.98$$

$$\frac{\overline{Z}}{\overline{Z}+4} = 0.95$$

We get $\overline{Z} = 33.0532$; $d = 0.4262$; $U = 3.5574$;

4. How to solve the model.

We proceed recursively.

Equations (5) assumes that the country is price taker on the imports and export goods markets

$$p = p^* + t$$

Let us assume that the wage rate w and the price of the non traded good q are known. Then we can compute recursively in a subprogram

with equations (1) and (6)

$$X = A^{1/(1-\alpha)} \left(\frac{\alpha}{w} \right)^{\alpha/(1-\alpha)}$$

$$L_X = \frac{\alpha X}{w}$$

with equations (2) and (7)

$$Y = B^{1/(1-\beta)} \left[\frac{\beta(p-s)}{w} \right]^{\beta/(1-\beta)}$$

$$L_Y = \frac{\beta(p-s)Y}{w}$$

with *e*quations (3) and (9)

$$N = C^{1/(1-\gamma)} \left(\frac{\gamma q}{w} \right)^{\gamma/(1-\gamma)}$$

$$L_N = \frac{\gamma q N}{w}$$

with equation (9)

$$Z = Y$$

with equation (14)

$$D_N = T q^{-\eta}$$

Then we set

$$R = X + p*Y + qN$$

We have

$$C_N = \frac{\bar{a}\bar{b}}{\bar{b} + b\left(\frac{q}{p}\right)^{\sigma} \frac{p*+at}{p}} \frac{R}{q}$$

Finally the subprogram gives the numerical values of the two following functions of w and q for any choice of the values of w and q.

$$E_1(w,q) \equiv L - L_X - L_Y - L_N$$
$$E_2(w,q) \equiv N - C_N - D_N$$

Equations (4) and (15) imposes

$$E_1(w,q) = 0$$
$$E_2(w,q) = 0$$

Then, we can compute

$$C_X = \frac{aqC_N}{\bar{a}}\left[1 + \frac{b}{\bar{b}}\left(\frac{q}{p}\right)^{\sigma}\right]$$

$$C_Y = \frac{b}{\bar{b}}\left(\frac{q}{p}\right)^{1+\sigma} C_N$$

$$U(C_X, C_Y, C_Z) = C_Z \bar{b}^{\bar{a}/\sigma}\left(\frac{aq}{\bar{a}}\right)^{a}\left[1 + \frac{b}{\bar{b}}\left(\frac{q}{p}\right)^{\sigma}\right]^{1+\bar{a}/\sigma} / \left[b^{1/(1+\sigma)} + \bar{b}^{1/(1+\sigma)}\right]^{\bar{a}(1+\sigma)/\sigma}$$

Thus, we have turned a system of 16 equations with 16 unknown variables into a system of 2 equations with 2 variables. This system can be solved by a standard algorithm (Jacobi, Gauss-Seidel or better Newton Raphson).

5. The programs

I created the working directory Chapter 4 and put my programs inside. There is a main script program called Main.m and a series of function program. In this chapter we will just use Calibration.m and Simulation.m.

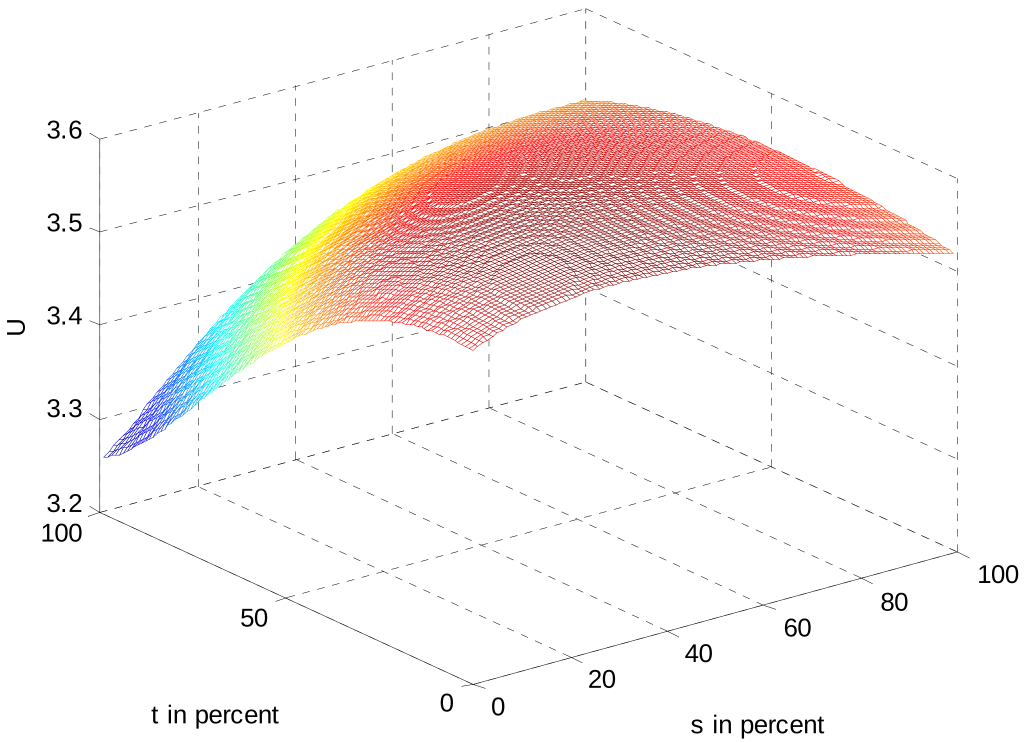
If we run a series of simulations we can compute the following table.

T	η	t^o (%)	s^o (%)	s^P	U^o	$U(0,0)$	Changes in U
0	3	0	11.50	11.50	3.5420	3.5364	0.16
1	3	15.21	16.12	13.34	3.5737	3.5574	0.46
1.5	3	18.55	16.97	13.89	3.5970	3.5731	0.67
0	1	0	11.50	11.50	3.5420	3.5364	0.16
1	1	26.22	18.63	14.36	3.5865	3.5574	0.82
1.5	1	39.25	21.05	15.93	3.6351	3.5772	1.62
0	0.5	0	11.50	11.50	3.5420	3.5364	0.16
1	0.5	34.59	20.28	15.12	3.5951	3.5574	1.06
1.5	0.5	65.49	24.49	18.43	3.6708	3.5786	2.58
0	0.3	0	11.50	11.50	3.5420	3.5364	0.16
1	0.3	41.06	21.40	15.71	3.6008	3.5574	1.22
1.5	0.3	109.2	26.97	22.44	3.7082	3.5792	3.60

U(0,0) represents utility when the tariff and the tax rates are zero. In the next chapter we will compute the optimal tariff and tax rate. When there is no tourism ($T = 0$), the optimal tariff is zero and the optimal pollution tax is equal to the Pigovian tax rate. When there is tourism ($T > 0$), the optimal tariff is positive and the optimal pollution tax is larger than the Pigovian tax rate.

The next graph represents the utility of households, including the effects of pollution, in function of the tariff rate and the pollution tax rate, when $T = 1$ and $\eta = 0.5$. We can see that the utility function is concave with respects to its to arguments. The table shows that the optimum is reached for $t = 34.59\%$ and $s = 20.28\%$.

The tariff, the tax rates and the gain in utility increase when the demand by tourists become less elastic, or when tourists' spending increase, which is unsurprising.



3.6
3.5
3.4
3.3
3.2
U
100
50
0
t in percent
0
20
40
60
80
100
s in percent